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A Generalization of the Cauchy-Hua Integral Formula on the Lie Ball

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Introduction.

First we fix notations and review known results following [3].

Put $\mathbf{E}=\mathbf{R}^{n+1}$ and $\tilde{\mathbf{E}}=\mathbf{C}^{n+1}$ $(n\geq 2)$. We put $z\cdot w=z_1w_1+z_2w_2+\cdots+z_{n+1}w_{n+1}$ for $z\in\tilde{\mathbf{E}}$ and $w\in\tilde{\mathbf{E}}$, $\|z\|^2=z\cdot\bar{z}$, and $z^2=z\cdot z$. We denote by

$$L(z)=\sqrt{\|z\|^2+\sqrt{\|z\|^4-|z^2|^2}}$$

the Lie norm of z. We have $|z^2|\leq\|z\|^2\leq L(z)^2$ for $z\in\tilde{\mathbf{E}}$. Let $r>0$. We denote by

$$\tilde{B}(r)=\{z\in\tilde{\mathbf{E}};\ L(z)<r\}$$

the open Lie ball of radius r, or the classical domain of type 4 (see [1]). $\tilde{B}[r]=\{z\in\tilde{\mathbf{E}};\ L(z)\leq r\}$ is called the closed Lie ball of radius r.

Let $\lambda\in\mathbf{C}$. The complex variety

$$\tilde{\mathbf{S}}_\lambda=\{z\in\tilde{\mathbf{E}};\ z^2=\lambda^2\}$$

is called the complex sphere of radius λ. We also call it the complex light cone if $\lambda=0$. Suppose $0<|\lambda|<r$ and put

$$\tilde{\mathbf{S}}_{\lambda,r}=\partial(\tilde{\mathbf{S}}_\lambda\cap\tilde{B}[r])=\{z\in\tilde{\mathbf{S}}_\lambda;\ L(z)=r\}\ , \tag{1}$$

$$\Sigma_{\lambda,r}=\{e^{i\theta}z;\ \theta\in\mathbf{R},\ z\in\tilde{\mathbf{S}}_{\lambda,r}\}\ . \tag{2}$$

$\tilde{\mathbf{S}}_{\lambda,r}$ and $\Sigma_{\lambda,r}$ are real analytic manifolds, $\dim_{\mathbf{R}}\tilde{\mathbf{S}}_{\lambda,r}=2n-1$, and $\dim_{\mathbf{R}}\Sigma_{\lambda,r}=2n$.

If $|\lambda|$ tends to r, then $\tilde{\mathbf{S}}_{\lambda,r}$ reduces to $\mathbf{S}_\lambda=\{\lambda x;\ x\in\mathbf{E},\ x^2=1\}$ the n-dimensional sphere of radius λ and $\Sigma_{\lambda,r}$ reduces to $\Sigma_r=\{e^{i\theta}x;\ \theta\in\mathbf{R},\ x\in\mathbf{S}_r\}$ the Lie sphere of radius r (see [2]). Note that $\dim_{\mathbf{R}}\mathbf{S}_\lambda=n$ and $\dim_{\mathbf{R}}\Sigma_\lambda=n+1$.

Let $\mathcal{O}(\tilde{B}(r))$ be the space of holomorphic functions on $\tilde{B}(r)$ and $\mathcal{O}(\tilde{B}[r])$ the space of germs of holomorphic functions on $\tilde{B}[r]$. We endow $\mathcal{O}(\tilde{B}(r))$ with the topology of uniform convergence on compact sets and $\mathcal{O}(\tilde{B}[r])$ with the locally convex inductive limit topology:

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