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Tunnel Number One Genus One Non-Simple Knots

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1. Introduction.

A knot in the 3-sphere is said to be *tunnel number one* if there exists an arc attached to the knot at its endpoints so that the complement of a regular neighbourhood of the resulting complex is a genus two handlebody. It is well-known that every torus knot and 2-bridge knot is tunnel number one. Although no composite knot is tunnel number one [7, 8], some non-simple knots are known to be tunnel number one [5]. It seems difficult to characterize tunnel number one knots.

Among prime knots, genus one knots are relatively easy to deal with and possess nice properties. For example, genus one fibred knots are exactly the trefoil knot and the figure-eight knot [1]. Any unknotting number one genus one knot is a double knot [4, 9].

In this note, as the first step for the determination of tunnel number one genus one knots, we shall completely determine all the tunnel number one genus one non-simple knots.

The tunnel number one non-simple knots in S^3 are classified by Morimoto and Sakuma [5]. We review it briefly.

Let K_0 be a non-trivial torus knot $T(p, q)$ of type (p, q) in S^3, and $L=K_1 \cup K_2$ a 2-bridge link $S(\alpha, \beta)$ of type (α, β) in S^3 with $\alpha \geq 4$. Then there is an orientation-preserving homeomorphism $f: E(K_2) \to N(K_0)$ which takes a meridian $m_2 \subset \partial E(K_2)$ of K_2 to a regular fibre $h \subset \partial N(K_0) = \partial E(K_0)$ of the Seifert fibration of $E(K_0)$. Here, for a complex C in S^3, $N(C)$ means the regular neighbourhood of C in S^3, and $E(C)$ means the exterior $S^3 - \operatorname{int} N(C)$. We denote the knot $f(K_1) \subset N(K_0) \subset S^3$ by $K(\alpha, \beta; p, q)$. Then $K(\alpha, \beta; p, q)$ is a tunnel number one non-simple knot, and conversely any tunnel number one non-simple knot is obtained in such a manner.

We will calculate the genera of $K(\alpha, \beta; p, q)$, so that we have the following.

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