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On p and q-Additive Functions

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1. Introduction.

Let q be an integer greater than 1. Let $a(n)$ be a complex-valued arithmetical function. $a(n)$ is said to be *q-additive* if

$$a(n)=\sum_{i\geq 0} a(b_i q^i)$$

for any positive integer $n=\sum_{i\geq 0} b_i q^i$ with $b_i \in \{0, 1, \cdots, q-1\}$, and $a(0)=0$. It follows from the definition that $a(n)$ is q-additive if and only if

$$a(nq^k+r)=a(nq^k)+a(r)$$

for any integer $n\geq 0$ and $k\geq 0$ with $0\leq r<q^k$. $a(n)$ is said to be *q-multiplicative* if

$$a(n)=\prod_{i\geq 0} a(b_i q^i)$$

for any positive integer n as above, and $a(0)=1$. $a(n)$ is a q-multiplicative function if and only if

$$a(nq^k+r)=a(nq^k)a(r)$$

for any $n\geq 0$ and $k\geq 0$ with $0\leq r<q^k$. If q-additive or q-multiplicative function $a(n)$ satisfies

$$a(bq^i)=a(b) \qquad (b\in\{0, 1, \cdots, q-1\}, i\geq 0)\,, \tag{1}$$

then $a(n)$ is said to be *strongly q-additive* or *strongly q-multiplicative*, respectively. We say $a(n)$ is p and q-additive if it is p-additive and also q-additive. Similarly, a p and q-multiplicative function is defined. The notion of q-additive functions and q-multiplicative functions were introduced by Gel'fond [2] and Delange [1] respectively and has been investigated by many authors (eg. [3], [4], [5]).

If $a(n)$ is a q-additive or q-multiplicative function, $a(n)$ is q^l-additive or q^l-

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