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On Characteristic Forms of Holomorphic Foliations

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1. Introduction.

Let M be a complex manifold of dimension n ($n \geq 2$) and $\mathscr{F}$ a holomorphic foliation on M of codimension q ($q \geq 1$). We denote the normal bundle of $\mathscr{F}$ by $\nu(\mathscr{F})$, and its dual by $\nu(\mathscr{F})^*$. Then we can define the obstruction $P_{\mathscr{F}} \in H^1(M, \nu(\mathscr{F})^* \otimes End(\nu(\mathscr{F})))$ to the existence of holomorphic projective connection $\pi = \{p_\alpha\}$ of $\nu(\mathscr{F})$. As is well-known, there always exists a C^∞ affine connection $a = \{a_\alpha\}$ of $\nu(\mathscr{F})$, by which we can define the Chern forms $\{c_k(a)\}_{k=1}^q$ of $\nu(\mathscr{F})$. Similarly there always exists a C^∞ (normal reduced) projective connection $\pi = \{p_\alpha\}$ of $\nu(\mathscr{F})$, and this defines a kind of C^∞ characteristic forms $\{P_k(\pi)\}_{k=1}^q$ of $\nu(\mathscr{F})$, which we call projective Weyl forms.

In this paper, we shall show that, for any C^∞ normal reduced projective connection $\pi = \{p_\alpha\}$ of $\nu(\mathscr{F})$, the projective Weyl forms are d-closed, and that there exists a C^∞ affine connection $a = \{a_\alpha\}$ of $\nu(\mathscr{F})$ which satisfies the following formulae;

$$\sum_{k=0}^{q} c_k(a) t^k = \frac{(1+\alpha t)^{q+1}}{1+(\alpha-\beta)t} \sum_{k=0}^{q} P_k(\pi) \left(\frac{t}{1+\alpha t} \right)^k ,$$

$$\sum_{k=0}^{q} P_k(\pi) t^k = (1-\alpha t)^q (1-\beta t) \sum_{k=0}^{q} c_k(a) \left(\frac{t}{1-\alpha t} \right)^k ,$$

where $c_k(a)$ is the k-th Chern form defined by the affine connection a, and both α and β are d-closed 2-forms which represent the de Rham cohomology class $[\frac{1}{q+1} c_1(a)]$ (Theorem).

As a corollary to this theorem, in the cohomology class level, we get the formulae;

$$\sum_{k=0}^{q} [c_k(a)] t^k = (1+[\alpha]t)^{q+1} \sum_{k=0}^{q} [P_k(\pi)] \left(\frac{t}{1+[\alpha]t} \right)^k ,$$

$$\sum_{k=0}^{q} [P_k(\pi)] t^k = (1-[\alpha]t)^{q+1} \sum_{k=0}^{q} [c_k(a)] \left(\frac{t}{1-[\alpha]t} \right)^k .$$

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