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Solvability of Nonstationary Problems for Nonhomogeneous Incompressible Fluids and the Convergence with Vanishing Viscosity

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1. Introduction.

Let Ω be a bounded or unbounded domain in $\mathbf{R}^3$ with a smooth boundary S. We consider the system of equations

$$(1.1)\qquad \begin{cases} \rho_t + v\cdot\nabla\rho = 0\,, \\ \rho[v_t + (v\cdot\nabla)v] + \nabla p = \mu\Delta v + \rho f\,, \\ \operatorname{div} v = 0 \end{cases}$$

in $Q_T = \Omega\times[0, T]$, $T>0$, where $f(x, t)$ is a given vector field of external forces, while the density $\rho(x, t)$, the velocity vector $v(x, t)$ and the pressure $p(x, t)$ are the unknowns. The viscosity coefficient μ is assumed to be a nonnegative constant.

This paper consists of two parts. In the first part, Part 1, we solve (1.1) under the following initial-boundary conditions:

If $\mu>0$,

$$(1.2)\qquad \begin{cases} v|_{S_T} = 0\,, \\ \rho|_{t=0} = \rho_0(x)\,, \\ v|_{t=0} = v_0(x)\,, \end{cases}$$

and if $\mu=0$,

$$(1.3)\qquad \begin{cases} v\cdot n|_{S_T} = 0\,, \\ \rho|_{t=0} = \rho_0(x)\,, \\ v|_{t=0} = v_0(x)\,, \end{cases}$$

where n is the unit outward normal to S, and $S_T = S\times[0, T]$.

In the second part, Part 2, when $\Omega=\mathbf{R}^3$, we consider the Cauchy problem (1.1) and

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