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Strong Solutions of Two Dimensional Heat Convection Equations with Dissipating Terms

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1. Introduction.

In this paper, we consider the existence of the strong solutions for the initial boundary value problems and the periodic problems of heat convection equations with dissipative terms in time dependent domains. The "dissipative terms" represent the friction of the fluid.

Let $\Omega(t)$ be a bounded domain in $\mathbf{R}^N$ with smooth boundary $\Gamma(t)$ for each $t\in[0, T]$, T be any positive number.

We consider the following heat convection equations in the noncylindrical domain $Q=\bigcup_{0\le t\le T}\Omega(t)\times\{t\}$ with lateral boundary $\Gamma=\bigcup_{0\le t\le T}\Gamma(t)\times\{t\}$.

$$\mathbf{u}_t-\nu\Delta\mathbf{u}+(\mathbf{u}\cdot\nabla)\mathbf{u}=-\frac{1}{\rho}\nabla p+\{1-\eta(\theta-d)\}\mathbf{g}+\mathbf{f}_1 \qquad (x, t)\in Q\,, \tag{1.1}$$

$$\operatorname{div}\mathbf{u}=0 \qquad (x, t)\in Q\,, \tag{1.2}$$

$$\theta_t-\kappa\Delta\theta+(\mathbf{u}\cdot\nabla)\theta+\frac{\nu}{2}D[\mathbf{u}]=f_2 \qquad (x, t)\in Q\,, \tag{1.3}$$

$$\mathbf{u}(x, t)=\mathbf{a}(x, t)\,,\quad \theta(x, t)=b(x, t) \qquad (x, t)\in\Gamma\,, \tag{1.4}$$

$$\mathbf{u}(x, 0)=\mathbf{u}_0(x)\,,\quad \theta(x, 0)=\theta_0(x) \qquad x\in\Omega(0)\,, \tag{1.5}$$

$$\mathbf{u}(x, 0)=\mathbf{a}(x, T)\,,\quad \theta(x, 0)=b(x, T) \qquad x\in\Omega(0)=\Omega(T)\,, \tag{1.6}$$

where

$$(\mathbf{u}\cdot\nabla)=\sum_{j=1}^{N}u^j\frac{\partial}{\partial x_j} \quad\text{and}\quad D[\mathbf{u}]=\sum_{i,j=1}^{N}\left(\frac{\partial u^i}{\partial x_j}+\frac{\partial u^j}{\partial x_i}\right)^2.$$

Unknown functions $\mathbf{u}=(u^1, u^2, \cdots, u^N)$, p and θ are the solenoidal velocity, pressure

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