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The Integral Representations of Harmonic Polynomials in the Case of $\mathfrak{sp}(p, 1)$

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Introduction.

Let $\mathfrak{g}$ be a complex reductive Lie algebra and let $\mathfrak{g}=\mathfrak{k}+\mathfrak{p}$ be the complexification of a Cartan decomposition of $\mathfrak{g}_{\mathbf{R}}=\mathfrak{k}_{\mathbf{R}}+\mathfrak{p}_{\mathbf{R}}$, where $\mathfrak{g}_{\mathbf{R}}$ is a noncompact real form of $\mathfrak{g}$.

Kostant-Rallis [3] stated some results on harmonic polynomials on $\mathfrak{p}$. On the other hand, for classical harmonic polynomials on $\mathbf{C}^p$, there have been many studies. It is well known that harmonic functions on $\mathbf{C}^p$ are represented by integrals on the unit sphere S^{p-1} or on some other $O(p)$-orbits and reproducing kernels of these formulas are expressed by Legendre polynomials (see, [2], [4], [5], [6], [7], [10], [12], [15], etc.).

In our previous paper [14] we obtained explicit integral representation formulas of harmonic polynomials in the case $\mathfrak{g}_{\mathbf{R}}=\mathfrak{su}(p, 1)$. From the Lie algebraic viewpoint, classical harmonic functions on $\mathbf{C}^p$ corresponds to harmonic functions on $\mathfrak{p}$ for the case $\mathfrak{g}_{\mathbf{R}}=\mathfrak{so}(p, 1)$. Therefore, integral representation formulas of harmonic polynomials in this case are known.

Our purpose of this paper is to obtain explicit integral representation formulas of harmonic polynomials and reproducing kernel of these formulas in remaining classical real rank one case, i.e. the case $\mathfrak{g}_{\mathbf{R}}=\mathfrak{sp}(p, 1)$. Our main results in this paper are described in Theorem 2.2, in which harmonic polynomials on $\mathfrak{p}$ for $\mathfrak{g}_{\mathbf{R}}=\mathfrak{sp}(p, 1)$ are represented by an integral on some $K_{\mathbf{R}}$-orbits. This result is similar to the cases $\mathfrak{g}_{\mathbf{R}}=\mathfrak{so}(p, 1)$ and $\mathfrak{g}_{\mathbf{R}}=\mathfrak{su}(p, 1)$.

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1. Preliminaries.

In this section we fix notations and review known results. For details, see [2], [3], [5], [7] and [16].

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