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Presheaves Associated to Modules over Subrings of Dedekind Domains

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Introduction.

Let A be a commutative ring with unity. For a subset E of $\mathrm{Spec}A$, we put

$$S_E=\bigcap_{\mathfrak{p}\in E}(A\backslash\mathfrak{p}) \quad (S_\phi=A)\,. \tag{1}$$

Then S_E is a saturated multiplicatively closed set.

To an A-module M, we associate a presheaf $\bar{M}$ in the following way. By putting

$$\bar{M}(U)=S_U^{-1}M \tag{2}$$

for an open subset U of $\mathrm{Spec}A$, we define a presheaf $\bar{M}$ of modules on $\mathrm{Spec}A$. Here $\bar{M}$ is not a sheaf in general. But the sheafification of $\bar{M}$ turns out to be the quasi-coherent $\tilde{A}$-module $\tilde{M}$. Then we ask the question: When is the presheaf $\bar{M}$ actually a sheaf?

Noting that $\bar{M}$ is a sheaf if and only if $\bar{M}=\tilde{M}$, we introduce the following three conditions for a ring A:

(S.1) $\bar{M}=\tilde{M}$ for any A-module M.

(S.2) $\bar{\mathfrak{a}}=\tilde{\mathfrak{a}}$ for any ideal $\mathfrak{a}$ of A.

(S.3) $\bar{A}=\tilde{A}$.

In the previous paper, the following facts are shown (see [5]):

FACT 1. *Suppose that A is a valuation ring. Then*
(i) *A satisfies the condition* (S.3).
(ii) (S.1)$\Leftrightarrow$(S.2)$\Leftrightarrow$$\mathrm{Spec}A$ *is a noetherian topological space.*

FACT 2. *Let A be a Dedekind domain. Then*

(S.1)$\Leftrightarrow$(S.2)$\Leftrightarrow$(S.3)$\Leftrightarrow$ *the ideal class group of A is torsion.*

FACT 3. *Suppose that A is a unique factorization domain. Then*
(i) *A satisfies the condition* (S.3).

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