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# Laguerre Character Sums

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## 1. Introduction.

Evans [1] defined character sum analogues of orthogonal polynomials and studied the properties of Hermite character sums. Greene [2] conducted a similar study of character sum analogues for hypergeometric series. Sawabe [7] considered the properties of Legendre character sums and showed the relation between Legendre character sums and hypergeometric character sums.

In this paper, we consider the properties of Laguerre character sums as defined by Evans [1]. Greene [3] showed that hypergeometric functions over finite fields arise as matrix elements of the principal series representations for $SL(2, q)$. We show that Laguerre character sums arise as matrix elements of a certain representation for a certain group, and derive two more properties of Laguerre character sums by making use of this result.

Throughout this paper we will use the following notations. Let $q$ be a positive integral power of an odd prime $p$. The finite field of $q$ elements is denoted by $\mathbf{F}_q$. The multiplicative group of $\mathbf{F}_q$ is denoted by $\mathbf{F}_q^\times$. The sets of the multiplicative and additive characters on $\mathbf{F}_q$ are denoted by $\hat{\mathbf{F}}_q^\times$ and $\hat{\mathbf{F}}_q^+$ respectively. The capital letters $A$, $B$, $C$, $M$, $N$, $\cdots$ and $\chi$ will denote multiplicative characters on $\mathbf{F}_q$. The trivial character will be denoted by 1 or $\varepsilon$, and the quadratic character by $\phi$. All multiplicative characters on $\mathbf{F}_q$ are defined to be zero at the zero element of $\mathbf{F}_q$. Define $\bar{N}$ by $N\bar{N}=1$. The letters $\lambda$, $\mu$ are reserved for additive characters on $\mathbf{F}_q$. For $x \in \mathbf{F}_q$, $\lambda_0(x)$ or $\zeta^x$ is defined by $\exp\left(\frac{2\pi i}{p}\mathrm{Tr}_{\mathbf{F}_q/\mathbf{F}_p}(x)\right)$, where $\mathrm{Tr}_{\mathbf{F}_q/\mathbf{F}_p}(x)$ denotes the trace of $x$ from $\mathbf{F}_q$ to $\mathbf{F}_p$. $\lambda_0$ is a non-trivial additive character on $\mathbf{F}_q$. For $w \in \mathbf{F}_q^\times$ and $\lambda \in \hat{\mathbf{F}}_q^+$, ${}^{w}\lambda$ denotes an additive character on $\mathbf{F}_q$ defined by ${}^{w}\lambda(a)=\lambda(wa)$ for $a \in \mathbf{F}_q$. For any non-trivial additive character $\mu$ on $\mathbf{F}_q$, there exists an unique element $w$ in $\mathbf{F}_q^\times$ such that $\mu={}^{w}\lambda_0$. We define a function $\delta$ on elements of $\mathbf{F}_q$ by

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