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Formulae for Relating the Modular Invariants and Defining Equations of $X_0(40)$ and $X_0(48)$

Takeshi HIBINO

Waseda University

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1. Introduction.

Let N be a positive integer and let $X_0(N)$ be the modular curve over $\mathbf{Q}$ associated to the modular group $\Gamma_0(N)$. As a defining equation of $X_0(N)$, we have the modular equation of level N, which is written in the following form:

$$F_N(j, j_N)=0\ , \quad F_N(S, T)\in \mathbf{Z}[S, T]\ ,$$

where $j=j(z)$ is the modular invariant, $j_N=j_N(z)=j(Nz)$, and z is the natural coordinate on $\mathscr{H}=\{z\in\mathbf{C}\,|\,Im(z)>0\}$. This equation has many useful properties, but its degree and coefficients are too large to be applied to practical calculations on $X_0(N)$. In the case of a hyperelliptic modular curve, its more manageable defining equation, which we call the normal form of the hyperelliptic curve, has been given by N. Murabayashi ([7]) and M. Shimura ([11]). In particular, for a hyperelliptic curve of the type $X_0(N)$, T. Hibino and N. Murabayashi ([4]) found a certain relation between the modular equation of level N and its normal form except for $N=40$, 48. The relation gives a formula expressing j in terms of the functions x, y on $X_0(N)$ which satisfy the normal form $y^2=f(x)$, $f(T)\in\mathbf{Q}[T]$.

In this paper, we deal with the remaining cases to complete our task. To be specific, for the defining equations $y^2=x^8+8x^6-2x^4+8x^2+1$, $y^2=x^8+14x^4+1$ for $N=40$, 48, respectively, we give formulae expressing j in terms of these x, y (Theorems 4.1, 4.2).

2. Basic idea for expressing j.

In the following, we sketch our idea ([4]) which is based on the computation of the Fourier coefficients of some modular forms (cf. [3], [5], [9], [12]). Let $\mathrm{Aut}(X_0(N))$ be the group of automorphisms of $X_0(N)$ over $\mathbf{C}$. For a positive integer $d\neq 1$ dividing

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