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J-Holomorphic Curves of a 6-Dimensional Sphere

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1. Introduction.

It is well known that a 6-dimensional sphere S^6 can be considered as a homogeneous space $G_2/SU(3)$ where G_2 is the Lie group of automorphisms of the octonions $\mathbf{O}$. From this representation, we can define an almost Hermitian structrure $(J, \langle\ ,\ \rangle)$ on a 6-dimensional sphere by making use of the vector cross product of the octonions. Also it is known that the almost Hermitian structure of S^6 satisfy the nearly Kähler condition $((D_X J)X = 0)$ where D is the Riemannian connection of S^6 with respect to the canonical metric and X is a tangent vector of S^6. A submanifold M in an almost Hermitian manifold N is called an almost complex submanifold if each tangent space of M is invariant under the almost complex structure of N. Almost complex submanifolds of S^6 were studied by many authors, for example, K. Sekigawa ([Se]), J. Bolton et al. ([Bo1]), R. L. Bryant ([Br1]), F. Dillen et al. ([D-V-V]), and A. Gray ([G]). A. Gray proved that there exists no 4-dimenional almost complex submanifold of S^6. Hence the dimension of almost complex submanifold of S^6 is either 2 or 6. In particular, we call a 2-dimensional almost complex submanifold a J-holomorphic curve. R. L. Bryant ([Br1]) constructed superminimal J-holomorphic curves of any compact Riemann surface to S^6 by using twistor methods with respect to the G_2-moving frame. Also, J. Bolton et al. ([Bo1, 2]) constructed non-superminimal J-holomorphic curves of 2-dimensional tori to S^6 by using the soliton theory. Curvature properties of J-holomorphic curves of S^6 were studied by K. Sekigawa ([Se]) and F. Dillen et al. ([D-V-V]). In this paper, we unify their results about J-holomorphic curves, making use of G_2-moving frame methods by R. L. Bryant and a Lemma of Eschenburg et al. [E-G-T] (also see ([Ch])), and give some results of curvcature properties of a J-holomorphic curve of S^6. Also we give two partial differential equations with respect to the Gauss curvature and the third fundamental form, and we obtain some G_2 rigidity theorem of J-holomorphic curves of S^6, genus formula (which

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