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Remodeling a DS-diagram into one with E-cycle

Hiroshi IKEDA, Masakatsu YAMASHITA and Kazuo YOKOYAMA

Kobe University, Toyo University and Sophia University

1. Introduction

B. G. Casler constructed a standard spine for a 3-manifold with boundary from the polyhedral structure, in [1]. He stated there that two 3-manifolds are homeomorphic if and only if they have a standard spine in common. Standard spines form a good subclass of the spines of 3-manifolds. Later, in [7], Ishii found a better class of spines for closed 3-manifolds. He constructed a spine by making use of a flow on the manifold and called such a spine a flow-spine. Spines of a closed manifold are understood to be the usual ones of the manifold from which a small ball is removed. It is known that the flow-spine form a good subclass of the standard spines. In this paper, we exhibit an algorithm to deform a standard spine to a flow-spine in the given closed manifold by a combinatorial topological method. It is, however, hard to see directly whether a standard spine is a flow-spine or not. By DS-diagrams (see Definition 1.1), we get rid of the difficulty. It is known in [5] that any closed 3-manifold has a DS-diagram constructed from a standard spine. The flow-spines correspond to the DS-diagrams with E-cycle, see [4] and [8]. Thus the problem above can be translated into the remodeling problem of a DS-diagram into one with E-cycle (see Definition 2.2).

The main theorem of this paper can be stated as follows (see Definition 1.2 for the notion of DS-isomorphism).

THEOREM 1.1. *Any DS-diagram is DS-isomorphic to a DS-diagram with E-cycle.*

We prove this theorem by finding a DS-isomorphism to get a DS-diagram with E-cycle algorithmically.

Including the concept of DS-isomorphism, let us review briefly some of the definitions made in [4] through [8] to understand the theorem.

Consider a 2-sphere S^2 and a connected 3-regular graph G embedded in S^2. Let V_G be the set of vertices of G. Then G induces a natural structure of cell complex $K(G)$ on S^2; 0-cells are elements of V_G, 1-cells are the connected components of $G - V_G$ and 2-cells are the connected components of $S^2 - G$. For a definition of cell complexes, see for example, [9].

DEFINITION 1.1. A triple $\Delta = (S^2, G, f)$ is called a *DS-diagram* if

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