TOKYO J. MATH.
VOL. 23, NO. 1, 2000

Affine Locally Symmetric Structures and Finiteness Theorems for Einstein-Weyl Manifolds

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(Communicated by T. Nagano)

Introduction.

A Weyl manifold is a smooth manifold M equipped with a torsion free affine connection D and a conformal structure $[g]$ such that D preserves $[g]$, in other words, for a representative metric g within $[g]$ there is a $\omega_g \in \Omega^1(M)$ for which $Dg = \omega_g \otimes g$.

We assume $\dim M \geq 3$.

Since the affine connection D leaves the conformal structure $[g]$ invariant, D has the holonomy group contained in $CO(n)$, the conformal orthogonal group.

For a Weyl manifold $(M, D, [g])$ the Riemannian curvature tensor R^D and the Ricci tensor Ric^D are defined in terms of D. The Ricci tensor is, in general, not necessarily symmetric.

The skew-symmetric part of Ric^D coincides, up to constant, with the closed 2-form $d\omega_g$ (Lemma 1 in §1). This 2-form is an invariant of a Weyl manifold measuring the extent to which the manifold differs from a trivial Weyl manifold. Here a Weyl manifold is called trivial if the affine connection coincides with the Levi-Civita connection of a representative metric within $[g]$.

In the present paper we are interested in Weyl manifolds carrying a particular geometric structure, especially in geometry of affine locally symmetric Weyl manifolds and of Weyl manifolds whose Ricci tensor is proportional to the conformal structure.

We consider in the first part Weyl manifolds admitting a D-parallel tensor, especially Weyl manifolds with D-parallel curvature tensor. It is shown in §1 that if the 2-form $d\omega_g$ is D-parallel, then a Weyl manifold is called *locally trivial*, that is, $d\omega_g = 0$. As a direct consequence an affine locally symmetric Weyl manifold must be locally trivial, since R^D is D-parallel (see Theorem 2 in §1).

As Proposition 2 in §1 shows, the local triviality implies that an affine locally symmetric Weyl manifold $(M, D, [g])$ turns out to be locally Riemannian, that is, M has at each point an open neighborhood on which $Dg = 0$ for a certain local metric g in $[g]$.

Received May 11, 1998
* This paper is partially supported by the Ministry of Education and Culture, Japan.