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On the Schur Indices of the Irreducible Characters of Finite Unitary Groups

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Let q be a power of a fixed prime number p. Let G be the unitary group $U(n, q^2)$ of degree n with respect to a quadratic extension $\mathbf{F}_{q^2}/\mathbf{F}_q$ ($\mathbf{F}_{q^e}$ denotes a finite field with q^e elements). The character table of G can essentially be obtained from the character table of the general linear group $GL(n, q)$ by a simple formal change that q is everywhere replaced by $-q$ (Ennola conjecture [2]; V. Ennola [2], G. Lusztig and B. Srinivasan [10], R. Hotta and T. A. Springer [7], G. Lusztig, D. Kazhdan, N. Kawanaka [8]; for $n \leqq 5$, the character table of G had been calculated by Ennola ([2]: $n = 2, 3$) and S. Nozawa ([12, 13]: $n = 4, 5$)). The purpose of this paper is to give some results concerning the Schur indices of the irreducible characters of G.

In the following, if χ is a complex irreducible character of a finite group and F is a field of characteristic 0, then $F(\chi)$ will denote the field generated over F by the values of χ and $m_F(\chi)$ will denote the Schur index of χ with respect to F.

Let χ be any one of the irreducible characters of G. Then the following two results are known:

THEOREM A (R. Gow [5, Theorem A]). *We have* $m_{\mathbf{Q}}(\chi) \leqq 2$.

THEOREM B ([18, Theorem 3]). *For any prime number* $l \neq p$, *we have* $m_{\mathbf{Q}_l}(\chi) = 1$.

The local index $m_{\mathbf{R}}(\chi)$ can be calculated by the method of Frobenius and Schur (see, e.g., Feit [3, pp. 20–21]): put $\nu(\chi) = (1/|G|) \cdot \sum_{g \in G} \chi(g^2)$; then $\nu(\chi) = 1, 0$ or -1; if $\nu(\chi) = 1$ or 0, then $m_{\mathbf{R}}(\chi) = 1$ and if $\nu(\chi) = -1$, then $m_{\mathbf{R}}(\chi) = 2$. But I think that when n is large the actual practice of this method is difficult. Of course, if $\mathbf{R}(\chi) = \mathbf{C}$, then $m_{\mathbf{R}}(\chi) = 1$. In the remark at the end of §2 of this paper we shall give, in terms of Ennola's parametrization of the irreducible characters of G ([2]; see §1, 1.5), a necessary and sufficient condition subject for that $\mathbf{R}(\chi) = \mathbf{R}$ or $\mathbf{C}$.

As to the local index $m_{\mathbf{Q}_p}(\chi)$, the following fact is implicit in [16, §3] in the case where $p \neq 2$.

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