

## Structure of Ideals and Isomorphisms of $C^*$ -crossed Products by Single Homeomorphism

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### 1. Introduction.

Let  $\Sigma = (X, \sigma)$  be a topological dynamical system where  $X$  is a compact Hausdorff space and  $\sigma$  is a homeomorphism. We denote by  $\alpha$  the automorphism induced by  $\sigma$  on the algebra  $C(X)$  of all continuous functions on  $X$ , namely  $\alpha(f)(x) = f(\sigma^{-1}x)$ . Let  $A(\Sigma)$  be the associated transformation group  $C^*$ -algebra, that is, the  $C^*$ -crossed product with respect to  $\alpha$ , regarding it as an action of the integer group  $\mathbf{Z}$ . We call this algebra a homeomorphism  $C^*$ -algebra.

Now one of the main problems about the interplay between topological dynamical systems and  $C^*$ -algebras is to determine relations between two dynamical systems when their associated homeomorphism  $C^*$ -algebras are isomorphic with each other (General isomorphism problem). A more restrictive problem is to settle the relationship between dynamical systems when an isomorphism between those  $C^*$ -algebras keeps their distinguished subalgebras of continuous functions (Restricted isomorphism problem). In case of Cantor minimal systems, we now know full aspects of isomorphism problems due to recent remarkable presentations by T. Giordano, I. Putnam and C. Skau ([6], [7]). With their results, we have then recognized that contrary to the case of measurable dynamical systems there appears a serious gap (even for minimal systems) from general isomorphisms to restricted isomorphisms.

In the author's joint work [3] with M. Boyle, we have solved the latter problem in the best possible way. However, we have not been making so much progress towards the general isomorphism problem. On the other hand, there are isomorphism theorems proved for some limited classes of dynamical systems such as rotations, Denjoy homeomorphisms etc. ([11], [12]), in which no obstruction appears. In fact, we do not know in general how the obstruction between two types of isomorphisms appears, and whether or not it disappears in an elementary connected space as it has been shown to be the case mentioned above.