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Deformation of Current Lie Algebra with Type Euclidean Group

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Introduction.

Let E be a vector space of dimension n, we note $\langle\,,\rangle$ the non degenerate bilinear, symmetric form and the Lie algebra of orthogonal group

$$\theta(E)=\{M\in M_n(K)/\langle MX,Y\rangle+\langle X,MY\rangle=0;\forall X,Y\in E\}\,.$$

There is an isomorphism between $\theta(E)$ and $\wedge^2 E$ (see section 2).

We consider $\mathfrak{g}=\theta(E)|\times E$ the semi direct product of $\theta(E)$ by E with Lie group G and $\mathfrak{g}_F$ the current Lie algebra over a smooth manifold M with type G.

We note $H_{\mathrm{loc}}(\mathfrak{g}_F,\mathfrak{g}_F)$ the cohomology of the Chevalley local cochains of the adjoint representation of $\mathfrak{g}_F$.

Our main purpose in this paper is to compute the second group of local cohomology of $\mathfrak{g}_F$ and determine the local deformations associated to it.

1. Some notation and definitions.

Let $m=\dim M$, we denote by $\mathfrak{g}_\infty(m)$ the Lie algebra of formal power series in m indeterminates $t^1,\cdots,t^m$ with coefficients in $\mathfrak{g}$. We denote by $\sum_\alpha t^\alpha x_\alpha$ a generic element of $\mathfrak{g}_\infty(m)$; here $\alpha=(\alpha_1,\cdots,\alpha_m)\in\mathbf{N}^m$ is a multi-index, t^α is the monomial $(t^1)^{\alpha_1}\cdots(t^m)^{\alpha_m}$, we denote by $|\alpha|$ the weight of α, let $|\alpha|=\sum_{i=1}^m\alpha_i$.

The adjoint representation of $\mathfrak{g}$ induces a representation $(\sum_\alpha t^\alpha x_\alpha)\to ad_{x_o}$.

For multi-indices $\alpha_1,\cdots,\alpha_p$, we define the $(\alpha_1,\cdots,\alpha_p)$ component of a $\mathfrak{g}$-valued p-cochain C on $\mathfrak{g}_\infty(m)$ to be the p-linear map from $\mathfrak{g}^p$ into $\mathfrak{g}$:

$$C_{\alpha_1\cdots\alpha_p}(x_1,\cdots,x_p)=C(t^{\alpha_1}x_1,\cdots,t^{\alpha_p}x_p)\,.$$

The cochain C is said to be homogeneous of weight k if $\sum_{i=1}^p|\alpha_i|\neq k\Rightarrow C_{\alpha_1\cdots\alpha_p}=0$.

Denote by $\bar{\mathfrak{g}}_\infty(m)$ the ideal of formal power series in m indeterminates, without constant term and with coefficient in $\mathfrak{g}$. The space of homogeneous $\mathfrak{g}$-valued cochains of order k on

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