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# The Involutions of Compact Symmetric Spaces, V

Tadashi NAGANO and Makiko S. TANAKA

*Sophia University and Science University of Tokyo*

## 0. Introduction.

In this part, we will prove that the roots of a symmetric space $M$ defined with the curvature (by using the Jacobi equation) make a root system $R(M)$ and determine their multiplicities. Thus we reestablish the known facts in a more geometric way (including the fact that the roots of a simple Lie algebra make a root system). Conversely, $R^m(M)$, $R(M)$ with the information of the multiplicity, allows one to recover the curvature of $M$; that is, $R^m(M)$ is a simple and complete description of the curvature tensor (although we will not give a proof).

In our geometric method of determining the multiplicity, we use the classification of the Hopf fibrations by Adams in case the rank $r(M)$ is 1. In case $r(M) = 2$, we use that of the homogeneous isoparametric hypersurfaces of spheres [M2] (or, equivalently, that of the compact Lie groups $K \subset SO(n)$ acting linearly on $R^n$ whose principal orbits have codimen $= 2$ by Hsiang-Lawson [HL] and others). The results in these cases are by and large enough to finish the job in the other case of $r(M) > 2$. To complete the job, we have studied the (adjoint) action of $\boldsymbol{k}(0)$, the centralizer of $\boldsymbol{a}$ in $\boldsymbol{k}$, in 2.2, which will give a deeper isnsight into the symmetric space.

In the final section, we will characterize a (local) Kählerian symmetric space in terms of $R^m(M)$ (Theorem 4.1) and in terms of centriole (see 1.9) (Theorem 4.5).

SYMBOLS. The symmetric spaces are denoted by standard notations (as in [H]) with minor exceptions such as $G_p(K^n)$ denoting the Grassmann manifold of the $p$ dimensional subspaces of a linear space $K^n$ and $AI(n) := SU(n)/SO(n)$, etc. $\mathcal{L}G :=$ the Lie algebra of $G$. $\boldsymbol{m}^2 := [\boldsymbol{m}, \boldsymbol{m}]$ for a linear subspace of a Lie algebra. See [B] for the symbols and numberings of the roots and weights; in particular, $(\varepsilon_1, \cdots, \varepsilon_n)$ is an orthonormal system.

## 1. Basic concepts of symmetric spaces.

1.0 DEFINITION. A connected smooth manifold $M$ is *a symmetric space* if (1) there is assigned an involutive diffeomorphism $s_x : M \to M$ of which $x$ is an isolated fixed point and

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