TOKYO J. MATH.
VOL. 23, NO. 2, 2000

Spherical Harmonics on $U(n)/U(n-1)$ and Associated Hilbert Spaces of Analytic Functions

Shigeru WATANABE

The University of Aizu

(Communicated by T. Nagano)

1. Introduction.

V. Bargmann showed in [1] that a generating function for the system of the Hermite polynomials can be regarded as the integral kernel of a unitary mapping from an L^2 space onto a Hilbert space of analytic functions. Then we have the following problem: is a similar construction possible for any system of classical orthogonal polynomials? That is, for any system of orthogonal polynomials, can we construct its generating function which can be regarded as the integral kernel of a unitary mapping from an L^2 space onto a Hilbert space of analytic functions? This is also indicated in Bargmann's paper.

Let $\mathbf{R}$ or $\mathbf{C}$ be the field of real or complex numbers, $S(\mathbf{R}^n)$ or $S(\mathbf{C}^n)$ the unit sphere in $\mathbf{R}^n$ or $\mathbf{C}^n$ and $x \mapsto \bar{x}$ the usual conjugation in $\mathbf{C}$.

We denote by F the Hilbert space of analytic functions $f(w)$ of n complex variables $w = {}^t(w_1, w_2, \cdots, w_n) \in \mathbf{C}^n$, with the inner product defined by

$$(f, g) = \pi^{-n} \int_{C^n} \overline{f(w)} g(w) \exp(-|w_1|^2 - \cdots - |w_n|^2) dw_1 \cdots dw_n ,$$

where

$$dw_1 \cdots dw_n = du_1 \cdots du_n dv_1 \cdots dv_n , \quad w_j = u_j + iv_j \ (u_j, v_j \in \mathbf{R}) ,$$

and by H the usual Hilbert space $L^2(\mathbf{R}^n)$.

V. Bargmann constructed in [1] a unitary mapping A from H onto F given by an integral operator whose kernel is considered as a generating function of the Hermite polynomials. More precisely, $f = A\phi$ for $\phi \in H$ is defined by

$$f(w) = \int_{R^n} A(w, t)\phi(t) d^n t ,$$

where

$$A(w, t) = \pi^{-n/4} \prod_{j=1}^{n} \exp\left\{-\frac{1}{2}(w_j^2 + t_j^2) + 2^{1/2} w_j t_j\right\} .$$

Received December 18, 1998