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Expansivity, Pseudoleaf Tracing Property and Semistability of Foliations

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1. Introduction.

A foliation is viewed as a generalization of a nonsingular flow, and it is natural to expect that various notions and results in the theory of dynamical systems might also be valid for foliations. In fact, several important results have been obtained from this viewpoint: e.g., the Poincaré-Bendixson theory for C^2 foliations (by Cantwell-Conlon, Hector etc.), the theory of transverse invariant measures and foliation cycles (by Plante, Sullivan etc.). One of the recent works on these lines was done by Ghys, Langevin and Walczak [7, 12], who defined an entropy for a foliation and showed that the notion is effective to measure the qualitative complexity of foliations (see also [9] and [13]). In [10] we considered the expansivity for foliations and obtained a characterization of an expansive codimension-one foliation in terms of local minimal sets (see also [11, Section D]).

In this paper we introduce a notion called the pseudoleaf tracing property (PLTP for short) for foliations. As is well known, the pseudo-orbit tracing property (POTP) plays an important role in topological dynamics to prove stability results. Our PLTP is its natural analogue in foliation theory. We give a few examples of foliations with the PLTP, and generalize a fundamental stability theorem for dynamical systems to foliations:

THEOREM A. *Let M be a closed Riemannian manifold and $\mathcal{F}$ a smooth foliation on M. If $\mathcal{F}$ is expansive and has the PLTP, then $\mathcal{F}$ is semistable.*

In dynamical systems, the corresponding results are obtained by Walters [17] for homeomorphisms and by Thomas [14] for flows.

We also prove that the PLTP, the expansivity and the semistability are preserved under the suspension procedure:

THEOREM B. *Let $\Phi : \Gamma \to \mathrm{Diff}(X)$ be a smooth action of a finitely presented group Γ on a compact Riemannian manifold X, and let $\mathcal{F}_\Phi$ be a suspension foliation of Φ. Then,*

(1) *Φ has the POTP if and only if $\mathcal{F}_\Phi$ has the PLTP.*

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