

On Some Maximal Galois Coverings over Affine and Projective Planes II

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1. Introduction.

This note is the second part of the preceding paper [NT] which we refer as Part I in the following. We recall that in Part I the finite maximal Galois coverings over a complex affine plane \mathbf{A}^2 with branch locus $B_q := \{(v, w) \in \mathbf{A}^2 \mid w^2 = v^q\}$ with q odd were studied in some detail, and further, the existence of maximal Galois coverings over a complex projective plane \mathbf{P}^2 with branch locus $\overline{B_q} \cup l_\infty$ was discussed, where $\overline{B_q}$ is the projective closure of B_q and l_∞ is the infinite line.

In this note, we study the finite maximal Galois coverings over \mathbf{A}^2 (resp. \mathbf{P}^2) with branch locus B_q (resp. $\overline{B_q} \cup l_\infty$) with even q , which remained untouched in Part I.

Our main results are: (i) the maximal Galois covering of \mathbf{A}^2 with branch locus B_q exists and is isomorphic to \mathbf{A}^2 if the corresponding maximal Galois group is finite (Theorem 4) and (ii) a criterion for the existence of maximal Galois coverings over \mathbf{P}^2 with branch locus $\overline{B_q} \cup l_\infty$ (Theorem 7).

We note that the finite maximal Galois coverings of \mathbf{P}^2 thus obtained are all rational so that they will hopefully be good examples of rational normal projective surfaces which are finite maximal Galois coverings over \mathbf{P}^2 with simple branch loci and also with explicitly calculated Galois groups.

Our main results mentioned above are easy consequences of the explicit calculations of the maximal Galois groups with some simple presentation. We thus spent many pages on elementary combinatorial group-theoretic computation, which may be boring. The reason for our doing this is twofold: (i) these explicit descriptions of the maximal Galois groups are essentially used in our main results (ii) it will be convenient for the readers who are not accustomed to combinatorial group theory.

In order to calculate the order of finite groups with given presentation, we used Cayley/Magma system, which performs the Todd-Coxeter process on computers.