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A Note on the Exponential Diophantine Equation $a^x + db^y = c^z$

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1. Introduction.

Let $\mathbf{N}$ be the set of all positive integers. Let a, b, c be fixed positive integers, and let d be a fixed prime with $d \equiv 3 \pmod 8$. In [2], using a lower bound for linear forms in two logarithms due to Laurent, Mignotte and Nesterenko [1], Terai and Takakuwa proved that if a, b, c, d satisfy

(1) $$a^2 + db^2 = c^2, \quad a \equiv 3 \pmod 8, \quad 4 \parallel b, \quad \left(\frac{b}{a}\right) = -1$$

(2) $$a \geq \lambda b, \quad d < 23865310019,$$

where (*/*) denote the Jacobi symbol and

$$\lambda = \sqrt{d}\left(\exp\left(2\left(\frac{\log d + 2}{\log 5} + 3231\right)^{-1}\right) - 1\right)^{-1/2},$$

then the equation

(3) $$a^x + db^y = c^z, \quad x, y, z \in \mathbf{N},$$

has only the solution $(x, y, z) = (2, 2, 2)$. In this paper, by an elementary approach, we prove the following result.

THEOREM *If a, b, c, d satisfy* (1) *and*

(4) $$a = db_2^2 - b_1^2 \quad b = 2b_1b_2 \quad c = db_2^2 + b_1^2,$$

where b_1, b_2 are positive integers satisfying $b_1 > 1$, $b_1 \equiv 1 \pmod 4$, $2 \parallel b_2$ and $\gcd(b_1, b_2) = 1$, then (3) *has only the solution $(x, y, z) = (2, 2, 2)$.*

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