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Suspension Order of the Suspended Real 6-Projective Space

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1. Introduction.

In this note all spaces and homotopies are based. We denote by ι_X the homotopy class of the identity map of a space X. We denote by ΣX a suspension of X. The self-homotopy set $[\Sigma X, \Sigma X]$ is a group, called a track group ([1]). The order of $\iota_{\Sigma X}$ is called the suspension order of X ([8]). Let P^n be the real n-dimensional projective space. The author proved that the suspension order of $\Sigma^2\mathrm{P}^6$ is 8 ([6]). The purpose of this note is to show the following.

THEOREM 1.1. *The suspension order of* $\Sigma\mathrm{P}^6$ *is* 8.

As a direct consequence of this theorem, we have ([3])

COROLLARY 1.2. *The suspension order of* $\Sigma\mathrm{P}^{2n}$ *is* $2^{\varphi(2n)}$, *where* $\varphi(m)$ *stands for the number of integers* k *satisfying* $1 \le k \le m$ *and* $k \equiv 0, 1, 2$ *or* $4 \mod 8$.

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2. A review of the result of [6].

First we fix the notation. We denote by $i_{k,n} : \mathrm{P}^k \hookrightarrow \mathrm{P}^n$ for $k \le n$ the inclusion map and by $\gamma_n : S^n \to \mathrm{P}^n$ the covering map. We note that

$$i_{m,n} \circ i_{k,m} = i_{k,n} \quad \text{for } k \le m \le n \quad \text{and} \quad i_{k,n} \circ \gamma_k = 0 \quad \text{for } k < n. \tag{1}$$

We often use the same letter for a mapping and its homotopy class. We set $\iota_n = \iota_{S^n}$. Let $\eta_2 \in \pi_3(S^2)$ and $\nu_4 \in \pi_7(S^4)$ be the Hopf maps. We set $\eta_n = \Sigma^{n-2}\eta_2$ $(n \ge 2)$, $\eta_n^2 = \eta_n \circ \eta_{n+1}$ and $\nu_n = \Sigma^{n-4}\nu_4$ $(n \ge 4)$. We recall the following result about the 2-primary components of homotopy groups of spheres ([7]):

$$\pi_n(S^n) = \mathbf{Z}\{\iota_n\} \ (n \ge 1)\,, \quad \pi_3(S^2) = \mathbf{Z}\{\eta_2\}\,, \quad \pi_{n+1}(S^n) = \mathbf{Z}_2\{\eta_n\} \ (n \ge 3)\,,$$

$$\pi_{n+2}(S^n) = \mathbf{Z}_2\{\eta_n^2\} \ (n \ge 2)\,, \quad \pi_6(S^3) = \mathbf{Z}_4\{\nu'\}\,, \quad \pi_7(S^4) = \mathbf{Z}\{\nu_4\} \oplus \mathbf{Z}_4\{\Sigma\nu'\}\,,$$

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