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Between Lie Norm and Dual Lie Norm

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Introduction.

In August 1999, we discussed the double series expansion of holomorphic functions on the dual Lie ball ([2]). Looking at our results we conjectured that there was a series of norms between the Lie norm and the dual Lie norm.

The Lie norm $L(z)$ on $\mathbf{C}^n$ is defined by

$$L(z) = \sqrt{\|z\|^2 + \sqrt{\|z\|^4 - |z^2|^2}}\,, \tag{1}$$

where $\|z\|^2 = |z_1|^2 + |z_2|^2 + \cdots + |z_n|^2$, $\quad z^2 = z_1^2 + z_2^2 + \cdots + z_n^2$ for $z = (z_1, z_2, \cdots, z_n)$.

The dual Lie norm $L^*(z)$ is defined as follows: $L^*(z) = \sup\{|z \cdot \zeta|;\ L(\zeta) \leq 1\}$, where $z \cdot \zeta = z_1\zeta_1 + z_2\zeta_2 + \cdots + z_n\zeta_n$ for $z = (z_1, z_2, \cdots, z_n)$ and $\zeta = (\zeta_1, \zeta_2, \cdots, \zeta_n)$. $L^*(z)$ has the following expression:

$$L^*(z) = \sqrt{(\|z\|^2 + |z^2|)/2} = \frac{1}{2}\left(L(z) + \frac{|z^2|}{L(z)}\right).$$

Noting $|z^2|/L(z) = \sqrt{\|z\|^2 - \sqrt{\|z\|^4 - |z^2|^2}}$, we can write

$$L^*(z) = \frac{1}{2}\left(\sqrt{\|z\|^2 + \sqrt{\|z\|^4 - |z^2|^2}} + \sqrt{\|z\|^2 - \sqrt{\|z\|^4 - |z^2|^2}}\right) \tag{2}$$

(see [1] and [5]).

For $p \geq 1$, we define the function $N_p(z)$ on $\mathbf{C}^n$ as follows:

$$N_p(z) = \left\{\frac{1}{2}\left((\|z\|^2 + \sqrt{\|z\|^4 - |z^2|^2})^{p/2} + (\|z\|^2 - \sqrt{\|z\|^4 - |z^2|^2})^{p/2}\right)\right\}^{1/p}.$$

It is clear that $N_2(z)$ is equal to the Euclidean norm $\|z\|$. We have $N_1(z) = L^*(z)$ by (2) and $\lim_{p\to\infty} N_p(z) = L(z)$ by (1). If $n = 2$, then $N_p(z)$ is equivalent to the Lebesgue L^p norm (see (5)).

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