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On Some Modules Attached to the Lubin-Tate Formal Groups

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Introduction.

Let p be a prime number and denote by $\mathbf{Q}_p$, $\mathbf{Z}_p$ and $\mathbf{C}_p$, the p-adic rational number field, the ring of integers of $\mathbf{Q}_p$ and the completion of the algebraic closure of $\mathbf{Q}_p$, respectively. Let $\mathcal{O}$ denote the ring of integers of $\mathbf{C}_p$.

Let $F(X, Y)$ be a Lubin-Tate formal group over $\mathbf{Z}_p$ and $h(X)$ a meromorphic series in $\mathcal{O}((X))^{\times}$. In [12], Shiratani and Imada constructed a p-adic zeta function $\zeta_p(s, F, h)$, which explains many well-known p-adic interpolating functions in a unified manner. For example, if $F(X, Y)$ is the formal multiplicative group $\mathbf{G}_m(X, Y) = (X+1)(Y+1)-1$ and $h(X) = X$, then $\zeta_p(s, \mathbf{G}_m, X)$ is the ordinary p-adic zeta function. If F is the formal group associated with an elliptic curve over $\mathbf{Z}$ having complex multiplication with ordinary reduction, then $\zeta_p(s, F, X)$ coincides with the p-adic zeta function for the elliptic curve ([10]).

Let χ be a primitive Dirichlet character with conductor a power of p. In [7], under the slightly generalized situation that $F(X, Y)$ is a relative Lubin-Tate formal group defined over the ring of integers of an unramified extension of $\mathbf{Q}_p$, we constructed a meromorphic function $L_p(s, \chi, F, h)$, which is an extension of $\zeta_p(s, F, h)$. Especially, $L_p(s, \chi, \mathbf{G}_m, X)$ coincides with the Kubota-Leopoldt p-adic L-function $L_p(s, \chi)$.

As is well known, Iwasawa gave the fascinating result that $L_p(s, \chi)$ is closely related to the Galois structure of the local units modulo the closure of the cyclotomic units ([4], [9, Chapter 7], [15, Section 13.8]). This result was extended to abelian fields by Gillard, Tsuji and so on ([3], [13]). It is also well known that Coates and Wiles discovered the analogue of this result for the elliptic units ([1]).

The main purpose of this paper is to generalize the above result of Iwasawa for the function $L_p(s, \chi, F, h)$ under the situation that F is defined over $\mathbf{Z}_p$ and that $h(X)$ satisfies certain appropriate conditions (Theorems 4.2 and 4.3). For this purpose, we use the method of the logarithmic derivatives developed by Coates and Wiles [1]. Let us give a description of each section.

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