TOKYO J. MATH.
VOL. 24, NO. 2, 2001

On the Complex WKB Method for a Secondary Turning Point Problem

Minoru NAKANO

Keio University

1. Introduction.

1.1. We consider the following n-th order linear ordinary differential equation containing a small parameter ε

(1.1)

$$\varepsilon^{nh} y^{(n)} = \sum_{k=1}^{n} \varepsilon^{(n-k)h} p_k \cdot (x^m - \varepsilon^l)^k y^{(n-k)} \quad (x, y \in \mathbf{C}, |x| \le x_0, 0 < \varepsilon \le \varepsilon_0, ' := d/dx),$$

where h, x_0 and ε_0 are positive constants, and m and l are positive integers. The characteristic equation for (1.1) is defined by

$$L(x, \lambda) := \lambda^n - \sum_{k=1}^{n} p_k \cdot x^{km} \lambda^{n-k} = 0 \tag{1.2}$$

and we suppose that it is factored as follows:

$$L(x, \lambda) = \prod_{k=1}^{n} (\lambda - a_k \cdot x^m) = 0\,, \tag{1.3}$$

where a_k's are real constants such that

$$a_1 < a_2 < \cdots < a_n; \quad \forall a_k \neq 0\,. \tag{1.4}$$

Then the characteristic roots $\lambda_k := a_k \cdot x^m$ coincide at the origin $x = 0$, which is, by definition, *a turning point* of (1.1). Furthermore we suppose that the following equality among three constants h, m and l is valid:

$$h = \frac{l(m+1)}{m} + 1\,. \tag{1.5}$$

We call (1.5) *the singular perturbation condition*, by which *a stretched equation* (2.2) in §2 below becomes a singular perturbation type. The stretched equation possesses its own turning points and they are called *secondary turning points* of (1.1). Thus, we call our analysis *a secondary turning point problem.*

Received December 10, 1997
Revised May 10, 2001