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Abelian Number Fields Satisfying the Hilbert-Speiser Condition at $p = 2$ or 3

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1. Introduction

Let F be a number field and $\mathcal{O}_F$ the ring of integers of F. Let N/F be a finite Galois extension with group G. We say that N/F has a normal integral basis (NIB for short) when $\mathcal{O}_N$ is cyclic over the group ring $\mathcal{O}_F[G]$. Hilbert and Speiser proved that any finite tame abelian extension of the rationals $\mathbf{Q}$ has a NIB. Let p be a prime number. We say that F satisfies the condition (H_p) when any tame cyclic extension N/F of degree p has a NIB. As mentioned above, $\mathbf{Q}$ satisfies (H_p) for any prime number p. On the other hand, Greither *et al.*[4] proved that any number field $F \neq \mathbf{Q}$ does not satisfy (H_p) for infinitely many p. So, it is of interest to determine which number field satisfies (H_p) or not. All imaginary quadratic fields satisfying (H_2) were determined by Carter [1]. There are exactly 3 such fields. All quadratic fields satisfying (H_3) were determined by [1] and Ichimura [2], independently. There are exactly 12 such fields. The purpose of this paper is to determine all imaginary abelian fields satisfying (H_2) and all abelian fields satisfying (H_3). We obtained the following result.

THEOREM.
(I) *Among all imaginary abelian fields F with $[F : \mathbf{Q}] \geq 3$, there exist exactly* 14 *fields satisfying (H_2), which are given in Table* 1 *at the end of this paper.*
(II) *Among all abelian fields F with $[F : \mathbf{Q}] \geq 3$, there exist exactly* 15 *fields satisfying (H_3), which are given in Table* 2.

2. Lemmas

Let F be a number field. For an integer $a \in \mathcal{O}_F$, let $Cl_F(a)$ be the ray class group of F defined modulo the ideal $(a) = a\mathcal{O}_F$. In particular, $Cl_F = Cl_F(1)$ is the absolute class group

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