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Minimal Free Resolution of Curves of Degree 6 or Lower in the 3-Dimensional Projective Space

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Let $C \subset \mathbf{P}^3$ be a nondegenerate smooth irreducible space curve of degree d and genus g over an algebraically closed field K. It is called n-regular if

$$H^1(\mathbf{P}^3, \mathcal{I}(n-1)) = H^2(\mathbf{P}^3, \mathcal{I}(n-2)) = 0\,,$$

where $\mathcal{I}$ is the ideal sheaf of C. Let

$$0 \to \bigoplus_{k\geq 4} S^{c_k}(-k) \to \bigoplus_{j\geq 3} S^{b_j}(-j) \to \bigoplus_{i\geq 2} S^{a_i}(-i) \to I \to 0$$

be a minimal free resolution of the defining ideal

$$I = \bigoplus_{l\geq 0} H^0(\mathbf{P}^3, \mathcal{I}(l)) \subset S = K[x_0, x_1, x_2, x_3]\,.$$

If C is n-regular, then $a_i = b_j = c_k = 0$ for $i \geq n+1$, $j \geq n+2$ and $k \geq n+3$. (See [1].) We call $(a_2, \cdots, a_n \mid b_3, \cdots, b_{n+1} \mid c_4, \cdots, c_{n+2})$ the Betti sequence of C, which is the most important information of the minimal free resolution. When we fix degree and genus, the number of the Betti sequences is finite. But listing them is very difficult. See [2] when C lies in a smooth cubic surface. In this paper, we shall show the following.

THEOREM 1. *If $d \leq 6$, then the Betti sequence of C is as follows.*

(d, g)	*Betti sequence*	*type*			
(3, 0)	(3|2|0)	(1)*	(6, 0)	(1, 0, 0, 5|0, 0, 0, 8|0, 0, 0, 3)	(8)
(4, 0)	(1, 3|0, 4|0, 1)	(2)		(0, 1, 6|0, 0, 9|0, 0, 3)	(9)
(4, 1)	(2, 0|0, 1|0, 0)	(3)*		(0, 2, 2, 1|0, 0, 4, 2|0, 0, 1, 1)	(10)
(5, 0)	(1, 0, 4|0, 0, 6|0, 0, 2)	(4)	(6, 1)	(0, 2, 3|0, 0, 6|0, 0, 2)	(11)
	(0, 4, 1|0, 3, 2|0, 0, 1)	(5)	(6, 2)	(0, 3, 1|0, 1, 3|0, 0, 1)	(12)
(5, 1)	(0, 5|0, 5|0, 1)	(6)	(6, 3)	(1, 0, 3|0, 0, 4|0, 0, 1)	(13)
(5, 2)	(1, 2|0, 2|0, 0)	(7)*		(0, 4|0, 3|0, 0)	(14)*
			(6, 4)	(1, 1, 0|0, 0, 1|0, 0, 0)	(15)*

*Here * means S/I is Cohen-Macaulay ring.*

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