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Stability of Cubic 3-folds

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Introduction.

Hilbert's idea of *null forms* appeared again as the *stability* and plays an important role in constructing the moduli space and its compactification in Geometric Invariant Theory of Mumford [7]. By virtue of the numerical criterion, one can determine the stable objects explicitly. For example, Hilbert proved the following. (See [3, §19] and [8, p15].)

THEOREM. *Let S be a cubic surface in the projective space $\mathbf{P}^3$.*

(1) *S is stable if and only if it has only rational double points of type A_1.*

(2) *S is semi-stable if and only if it has only rational double points of type A_1 or A_2.*

(3) *The moduli of stable ones is compactified by adding one point corresponding to the semi-stable cubic $xyz + w^3 = 0$ with 3 A_2 singularities.*

The stability of quartic surfaces is studied by Shah [11]. In this paper applying the same criterion to cubic 3-folds, i.e. hypersurfaces of degree 3 in $\mathbf{P}^4$, we prove the following.

MAIN THEOREM. *Let X be a cubic 3-fold.*

(1) *X is stable if and only if it has only double points of type $A_n : v^2+w^2+x^2+y^{n+1} = 0$ with $n \leq 4$.*

(2) *X is semi-stable if and only if it has only double points of type A_n with $n \leq 5$, $D_4 : v^2 + w^2 + x^3 + y^3 = 0$ or $A_\infty : v^2 + w^2 + x^2 = 0$. And if a semi-stable cubic 3-fold has A_∞ singularity, then it is isomorphic to the secant 3-fold, that is, the secant variety of rational normal curve in $\mathbf{P}^4$.*

(3) *The moduli of stable ones is compactified by adding two components. One is isomorphic to $\mathbf{P}^1$ and the other is an isolated point corresponding to the semi-stable cubic 3-fold $xyz + v^3 + w^3 = 0$ with 3 D_4 singularities.*

We remark that Collino [1] studies the degeneration of intermediate Jacobians for a family of cubic 3-folds approaching to the secant 3-fold.

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