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On the Embedded Eigenvalues for the Self-Adjoint Operators with Singular Perturbations

Kazuo WATANABE

Gakushuin University

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1. Introduction and assumptions.

This paper is a continuation of [7]. That is, in the framework of the $\mathcal{H}_{-2}$-construction we consider a finite rank perturbation of a self-adjoint operator H_0 without assuming semi-boundedness for H_0. The $\mathcal{H}_{-2}$-construction has been developed by A. Kiselev and B. Simon [1], S. T. Kuroda and H. Nagatani [2], [3] and have been applied to Schrödinger operators with a singular perturbation by H. Nagatani [4] and S. Shimada [6].

In this paper we consider the embedded eigenvalues of H_T and the existence of the wave operator $W_\pm(H_0, H_T)$. We prepare some notations. Let $\mathcal{H}$ be a Hilbert space with the inner product $\langle\cdot,\cdot\rangle$, H_0 a self-adjoint operator in $\mathcal{H}$ and $R_0(z) = (H_0 - z)^{-1}$ ($\operatorname{Im} z \neq 0$). We put $\mathcal{H}_s := \{u \in \mathcal{H}; \|(|H_0| + 1)^{s/2}u\| < \infty\}$ for $s \geq 0$, and $\mathcal{H}_s := (\mathcal{H}_{-s})^*$ for $s < 0$. Remark that $\mathcal{H}_s \subset \mathcal{H} \subset \mathcal{H}_{-s}$ for $s \geq 0$. For simplicity we use the same symbol $\langle\cdot,\cdot\rangle$ for the dual coupling $\langle\cdot,\cdot\rangle_{s,-s}$ of $\mathcal{H}_s$ and $\mathcal{H}_{-s}$ ($s \in \mathbf{R}$), and regard the operator $R_0(z)$ with $\operatorname{Im} z \neq 0$ as the element of $\mathcal{L}(\mathcal{H},\mathcal{H}) \cap \mathcal{L}(\mathcal{H}_s, \mathcal{H}_{s+2})$ for $\operatorname{Im} z \neq 0$.

DEFINITION. Define

$$W(z) = W(z,i) = (z - i)R_0(z)R_0(i)$$

and the operator $R_T(z)$ in $\mathcal{H}$

$$R_T(z) = R_0(z) - R_0(z)(1 + TW(z))^{-1}TR_0(z), \quad \operatorname{Im} z \neq 0\,. \tag{1}$$

To define the self-adjoint operator H_T for $T \in \mathcal{L}(\mathcal{H}_2, \mathcal{H}_{-2})$ we use the following theorem (cf. [3]).

THEOREM 1.1 ([3]). *If $T \in \mathcal{L}(\mathcal{H}_2, \mathcal{H}_{-2})$ satisfies*

$$T - T^* = TW(-i,i)T^* = T^*W(-i,i)T\,, \tag{2}$$

$$u - TR_0(i)u = 0, \quad u \in \mathcal{H}_0 \Rightarrow u = 0\,, \tag{3}$$

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