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Multiplicative SK Invariants for G-Manifolds with Boundary

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0. Introduction

Let G be a finite abelian group. In this paper, a G-manifold means an unoriented compact smooth manifold (which may have boundary) together with a smooth action of G. Let T be a map for m-dimensional G-manifolds which takes its values in the ring $\mathbf{Z}$ of rational integers and is additive with respect to the disjoint union of G-manifolds. We call T a G-SK invariant if it is invariant under equivariant cuttings and pastings (Schneiden und Kleben in German) [5, 6, 9]. For example, χ^H given by $\chi^H(M) = \chi(M^H)$ for G-manifolds M is a G-SK invariant, where χ is the Euler characteristic, H is a subgroup of G and $M^H = \{x \in M \mid hx = x \text{ for any } h \in H\}$. Further suppose that T is defined for all G-manifolds with various dimensions. Then it is said to be multiplicative if $T(M \times N) = T(M) \cdot T(N)$ for any G-manifolds M and N. For example, the above χ^H is multiplicative.

The main object of this paper is to characterize a form of multiplicative G-SK invariants. In [1, 3], the author has discussed such a question in case where G is a cyclic group of finite order.

In Section 1, we describe the irreducible G-modules and G-slice types. These notions are needed in order to proceed with our argument.

In Section 2, we first introduce an SK group $SK_*^G(\partial)$ resulting from equivariant cuttings and pastings of G-manifolds. In [4, 8], Koshikawa and the author have studied its SK_*-module structure, where SK_* is an SK ring of closed manifolds (Proposition 2.2). A G-SK invariant T induces an additive homomorphism $SK_*^G(\partial) \to \mathbf{Z}$. For a slice type σ, let χ_σ be a G-SK invariant defined by $\chi_\sigma(M) = \chi(M_\sigma)$, where M_σ is a G-submanifold of M with slice types containing σ (Definition 2.5). Then, using these χ_σ, we have a basis of a free $\mathbf{Z}$-module $\mathcal{T}_*^G$ consisting of all G-SK invariants [2] (Proposition 2.8). Next we study a multiplicative G-SK invariant, which is considered to be a ring homomorphism $SK_*^G(\partial) \to \mathbf{Z}$. Such an invariant T is said to be of type $\langle G/H \rangle$ if H is the minimum element (with respect to the inclusion of subgroups) in the set consisting of those subgroups K of G such that $T(G/K) \neq 0$ (Definition 2.11). For example, χ^H is of type $\langle G/H \rangle$. It is seen that T

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