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A New Proof for Some Relations among Axial Distances and Hook-Lengths

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Introduction

In this paper, we give a new proof of several relations among axial distances and products of hook-lengths using a generalized version of Lagrange's interpolation polynomial (Theorem 0.1). Applying this method to another case, a new relation is obtained (Theorem 0.2).

To state the main theorems, we introduce some terminology and notation following the books [4, 5]. Let $\lambda = (\lambda_1, \lambda_2, \dots, \lambda_l)$ be a partition of n. The *Young diagram* or *shape* of λ is an array of n boxes having l left-justified rows with row i containing λ_i boxes for $1 \leq i \leq l$. The box in row i and column j has coordinates (i, j), as in a matrix. The *conjugate* of a partition λ is the partition λ^* whose diagram is the transpose of the diagram λ. More precisely, for λ, $\lambda^* = (\lambda_1^*, \lambda_2^*, \dots, \lambda_{l'}^*)$ is defined by

$$\lambda_j^* = |\{k; \lambda_k \geq j\}| \quad (j = 1, 2, \dots, l' = \lambda_1)\,.$$

The *hook-length* of λ at $(i, j) \in \lambda$ is denoted by $h_\lambda(i, j)$ and defined by

$$h_\lambda(i, j) = \lambda_i + \lambda_j^* - i - j + 1\,.$$

For a partition λ, let $h(\lambda)$ denote the product of all the hook-lengths in λ, namely

$$h(\lambda) = \prod_{(i,j)\in\lambda} h_\lambda(i, j)\,.$$

For an indeterminate q, we define the *q-integer* $[i]$ by

$$[0] = 0 \quad \text{and} \quad [i] = \frac{q^i - q^{-i}}{q - q^{-1}} = q^{i-1} + q^{i-3} + \cdots + q^{-i+1} \ (i \geq 1)\,.$$

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