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Iwasawa Theory for Extensions with Restricted p-Ramification

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1. Introduction

Let p be a prime number. For an algebraic number field K of finite degree, consider an (arbitrary) subset T of $P(K)$, where $P(K)$ is the set of all primes above p of K:

$$T \subset P(K).$$

Let K_∞ be the cyclotomic $\mathbf{Z}_p$-extension of K and $T_\infty \subset P(K_\infty)$ the set of primes above T of K_∞. Then, by $\mathcal{M}_{T_\infty}(K_\infty)$, we denote the maximal abelian p-extension of K_∞ unramified outside T_∞. We call such an extension "the extension with restricted p-ramification".

Since $\Gamma := \mathrm{Gal}(K_\infty/K)$ acts on the Galois group

$$\mathcal{Y}_{T_\infty}(K_\infty) := \mathrm{Gal}(\mathcal{M}_{T_\infty}(K_\infty)/K_\infty)$$

by conjugation, it is regarded as a module over the power series ring $\Lambda := \mathbf{Z}_p[[T]]$ in the usual manner. This is finitely generated over Λ.

In this article, we investigate the following question: What are the Λ-rank of $\mathcal{Y}_{T_\infty}(K_\infty)$ and the μ-invariant of its Λ-torsion part $\mu(\mathcal{Y}_{T_\infty}(K_\infty)_{\Lambda-tor})$?

When $T = \varnothing$ (empty set), it is well known that $\mathcal{Y}_\varnothing(K_\infty)$ has Λ-rank zero by a result of Iwasawa and that it is conjectured that its μ-invariant vanishes. This is verified when K is an abelian field by Ferrero and Washington [FeWa]. It is also known that $\mathrm{rank}_\Lambda(\mathcal{Y}_{T_\infty}(K_\infty)) = r_2$ if $T = P(K)$, where r_2 is the number of complex primes of K. The μ-invariant of the Λ-torsion part of $\mathcal{Y}_{T_\infty}(K_\infty)$ is also conjectured to be zero and proved if K is abelian.

In case of CM-fields, the answer to the above question is known completely (cf. [JaMa]. See also Theorem 4.5 below).

On the other hand, for a general base field K and $T \subset P(K)$, we have a trivial lower bound of the Λ-rank (Proposition 2.3):

$$\mathrm{rank}_\Lambda(\mathcal{Y}_{T_\infty}(K_\infty)) \geq r_2 - \sum_{v \in P(K)-T} [K_v : \mathbf{Q}_p].$$

However, we do not know how the Λ-rank should be in general. We give the following partial result by applying the methods of Ax and Brumer.

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