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On the Descriptions of $\mathbf{Z}/p^2\mathbf{Z}$-Torsors by the Kummer-Artin-Schreier-Witt Theory

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Introduction

The Kummer-Artin-Schreier-Witt theory is the unified theory of the Kummer theory and the Artin-Schreier-Witt theory. We denote by p a prime number and ζ_n a primitive p^n-th root of unity such that ${\zeta_n}^p = \zeta_{n-1}$. Let $A = \mathbf{Z}_{(p)}[\zeta_n]$. The Kummer-Artin-Schreier-Witt sequence

$$0 \longrightarrow (\mathbf{Z}/p^n\mathbf{Z})_A \xrightarrow{i_n} \mathcal{W}_n \xrightarrow{\Psi^n} \mathcal{V}_n \longrightarrow 0$$

has the Artin-Schreier-Witt sequence as the special fiber and the Kummer type sequence as the generic fiber, where $\mathcal{W}_n$ and $\mathcal{V}_n$ are group schemes related to deformations of the additive group scheme to the multiplicative group scheme (cf. Section 2). This sequence is a key of the Kummer-Artin-Schreier-Witt theory. The case $n = 1$ of this theory (the Kummer-Artin-Schreier theory) was presented by Waterhouse [10] and Sekiguchi-Oort-Suwa [3] independently. In the general case, this theory was formulated by Sekiguchi-Suwa [5], [8] and [7].

Let X be a scheme, G a flat group scheme locally of finite type over X and X' a scheme over X such that G acts on X'. The scheme X' is a G-torsor over X if X' is locally isomorphic to G for the flat topology on X. In particular, if G is a finite group scheme, a G-torsor is a Galois G-extension. Now let $\mathrm{PHS}(G/X)$ be the set of all isomorphism classes of G-torsors over X. If G is a commutative affine group scheme over X, then $\mathrm{PHS}(G/X) \xrightarrow{\sim} \check{H}^1_{\mathrm{fl}}(X, G) \xrightarrow{\sim} H^1_{\mathrm{fl}}(X, G)$ (cf. Raynaud [2]). Therefore we can calculate torsors by the cohomology theory.

Our aim of this article is to give concrete descriptions of $\mathbf{Z}/p^2\mathbf{Z}$-torsors over an A-scheme X, that is to say, unramified cyclic coverings of degree p^2 over an A-scheme X. In order to give them, we use arguments similar to those using in the Kummer theory and the Artin-Schreier-Witt theory (cf. Section 1). Our main result is as follows:

ASSERTION 1 (cf. Section 3, 3.3). *Let X be an A-scheme, $\mathcal{U} = \{U_j\}$ an affine open covering on X. Let $f_{ij} \in Z^1(\mathcal{U}, \mathcal{W}_2)$ be a 1-cocycle such that $\Psi^2([f_{ij}]) = 0$. Then, if*

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