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On a Characteristic Function of the Tensor K-module of Inner Type Noncompact Real Simple Groups

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1. Introduction

Let $\mathbf{C}$ (resp. $\mathbf{R}$) denote the complex (resp. real) number field. We consider a connected simply connected complex simple Lie group $G_{\mathbf{C}}$ and a connected noncompact inner type simple real form G of $G_{\mathbf{C}}$. Let K be a maximal compact subgroup of G. We denote the Lie algebras of G and K respectively by $\mathfrak{g}$ and $\mathfrak{k}$. Let θ be the Cartan involution of $\mathfrak{g}$ corresponding to $\mathfrak{k}$. Let's denote the eigensubspace of θ of $\mathfrak{g}$ with the eigenvalue -1 by $\mathfrak{p}$. Then we have a Cartan decomposition: $\mathfrak{g} = \mathfrak{k} \oplus \mathfrak{p}$. Consequently the Lie algebra $\mathfrak{g}_{\mathbf{C}}$ of $G_{\mathbf{C}}$ is also decomposed by $\mathfrak{g}_{\mathbf{C}} = \mathfrak{k}_{\mathbf{C}} \oplus \mathfrak{p}_{\mathbf{C}}$, where $\mathfrak{k}_{\mathbf{C}}$ (resp. $\mathfrak{p}_{\mathbf{C}}$) is the complexification of $\mathfrak{k}$ (resp. $\mathfrak{p}$) in $\mathfrak{g}_{\mathbf{C}}$. Canonically K acts on the space $\mathfrak{p}_{\mathbf{C}}$. Let B be a maximal abelian subgroup of K. Since K is connected and G is an inner type simple Lie group, B is also a maximal abelian subgruop of G. Therefore B is a Cartan subgroup of G and K. Let $\mathfrak{b}_{\mathbf{C}}$ be the complexification of the Lie algebra $\mathfrak{b}$ of B. Let Σ be the root system of the pair $(\mathfrak{g}_{\mathbf{C}}, \mathfrak{b}_{\mathbf{C}})$. Then we have $\Sigma = \Sigma_K \cup \Sigma_n$, where Σ_K (resp. Σ_n) is the set of all compact (resp. noncompact) roots of Σ . We shall fix a positive root system P_K of Σ_K. Let (π_μ, V_μ) be a simple K-module with the highest weight μ. Then the tensor space $\mathfrak{p}_{\mathbf{C}} \otimes V_\mu$ is a unitary K-module. Let ν be a P_K-dominant integral form on $\mathfrak{b}_{\mathbf{C}}$ and V_ν a simple K-module corresponding to ν. We define a projection operator P_ν on $\mathfrak{p}_{\mathbf{C}} \otimes V_\mu$ by

$$P_\nu(Z) = \deg \pi_\nu \int_K kZ\overline{\operatorname{trace} \pi_\nu(k)} dk \quad \text{for } Z \text{ in } \mathfrak{p}_{\mathbf{C}} \otimes V_\mu ,$$

where dk is the Haar measure on K normalized as $\int_K dk = 1$. Let Γ_K be the set of all P_K-dominant integral form on $\mathfrak{b}_{\mathbf{C}}$. Then we have the following decomposition:

$$\mathfrak{p}_{\mathbf{C}} \otimes V_\mu = \oplus_{\omega \in \Sigma_n, \mu+\omega \in \Gamma_K} P_{\mu+\omega}(\mathfrak{p}_{\mathbf{C}} \otimes V_\mu) , \tag{1.1}$$

where $P_{\mu+\omega}(\mathfrak{p}_{\mathbf{C}} \otimes V_\mu) = \{0\}$ or is a simple K-module. The purpose of this paper is to characterize nontrivial K-module $P_{\mu+\omega}(\mathfrak{p}_{\mathbf{C}} \otimes V_\mu)$ by using a rational function. Let us state our results more precisely. We can prove that $P_{\mu+\omega}(\mathfrak{p}_{\mathbf{C}} \otimes V_\mu)$ is nontrivial if and only if

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