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Local Orbit Types of S-representations of Symmetric R-spaces

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Dedicated to Professor Katuhiro SHIOHAMA on his sixtieth birthday

E. Heintze and C. Olmos [HeiOlm] have investigated the local orbit types of s-representation of semisimple symmetric spaces in terms of restricted root systems. Their results have been generalized by H. Tamaru [Tama2]. But, as far as the author knows, there are no complete lists of all orbit types of s-representations of Riemannian symmetric spaces. The main purpose of this paper is to obtain a complete list of all local orbit types of s-representations of the following "symmetric R-spaces"; (i) the classical types of the rank 2: $T \cdot AI_2$, $T \cdot AII_2$, $AIII_2$, BDI_2, CI_2, CII_2, $CII_2 = Gr_2(\mathbf{H}^4)$, $DIII_2$, (ii) the classical types of the rank 3: $T \cdot AI_3$, $T \cdot AII_3$, $AIII_3$, BDI_3, CI_3, CII_3, $DIII_3$, (iii) the exceptional types: $EIII$, EIV, $EVII$, $FII = P^2(\mathbf{O})$; $G = G_2/SO(4)$ (as a normal space), (iv) the classical groups of the rank 2: $SO(4)$, $SO(5)$, $U(3)$, $Sp(2)$, (v) the classical groups of the rank 3: $SO(6)$, $SO(7)$, $U(4)$, $Sp(3)$, (vi) the real quadrics: $S^p \cdot S^q$ $(p \leq q)$, which is our main results (see Section 3). For a compact semisimple symmetric space, we get the result stated in Section 2 as follows;

THEOREM 0.1 (Criterion theorem 2.6 in Section 2). *Any two orbits of a compact semisimple symmetric space are locally diffeomorphic if and only if their closed subsystems in the restricted root system are conjugate.*

COROLLARY 0.2 (Corollary 2.8 in Section 2). *The number of the local orbit types of s-representations of a compact semisimple symmetric space is less than or equal to 2^r, where r is the rank of the symmetric space.*

Let $M = G/K$ be a compact semisimple symmetric space, where G is the identity component of the isometry group. Let H, H' be two points in the tangent space T_oM to M at the origin $o \in M$, and let K_H and $K_{H'}$ be the isotropy subgroups of K (identified with the linear isotropy group) at H and H', respectively. We denote by $\mathfrak{k}_H$ and $\mathfrak{k}_{H'}$ the Lie algebras of K_H and $K_{H'}$, respectively. We say that two orbits $K(H) = K/K_H$ and $K(H') = K/K_{H'}$ are of the *same orbit type* if K_H is conjugate to $K_{H'}$ in K under the automorphism group of K. Thus, we say that two orbits $K(H) = K/K_H$ and $K(H') = K/K_{H'}$ are of the *same local orbit type* if $\mathfrak{k}_H$ is conjugate to $\mathfrak{k}_{H'}$ in $\mathfrak{k}$ under the automorphism group of $\mathfrak{k}$. We say that

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