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Positive Foliations on Compact Complex Manifolds

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1. Introduction

We recall the following definition of singular meromorphic foliation.

DEFINITION 1.1. Let X be a smooth complex compact n-dimensional manifold. A dimension q, $1 \le q < n$, singular meromorphic foliation on X is defined by a rank q subsheaf E of the tangent bundle TX of X such that TX/E has no torsion and E is involutive, i.e. E is closed under Lie brackets.

If $q = 1$ (i.e. if the foliation is a foliation by curves) then the condition of involutiveness is automatically satisfied and E is a line bundle ([H], Prop. 1.9). The sheaf E is called the tangent sheaf to the leaves of the foliation. In this paper we study positive singular meromorphic foliations, mainly in the case in which X has a fibration.

DEFINITION 1.2. Let X be a smooth complex compact n-dimensional manifold and F a dimension q, $1 \le q < n$, singular meromorphic foliation on X defined by an exact sequence

$$0 \to E \to TX \to TX/E \to 0 \tag{1}$$

with E involutive, $\mathrm{rank}(E) = q$, and TX/E torsion free.

(a) We will say that F is *effective* or *non-negative* if the coherent sheaf E is generically spanned by its global sections, i.e. there is a non-empty open subset U of X such that the natural map $H^0(X, E) \otimes \boldsymbol{O}_X \to E$ is surjective at every point of U.

(b) We will say that F is *semi-positive* if we may take as U a Zariski open subset of X such that $X \setminus U$ is a closed analytic subset of X with codimension at least two.

(c) Assume $q = 1$. Then we will say that F is *strictly generically positive* if $h^0(X, E) \ge 2$ and the base locus of E has codimension at least two in X and we will say that E is *strictly generically effective* if $h^0(X, E) \ge 2$.

We need to use some kind of positivity for torsion-free coherent sheaves on compact complex manifolds.

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