TOKYO J. MATH.
VOL. 26, NO. 2, 2003

On an Airy Function of Two Variables II

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(Communicated by A. Tani)

0. Introduction

Consider an integral of the form

$$z_C(x, y) = \int_C \exp\Big(-\frac{t^4}{4} + \frac{xt^2}{2} + yt\Big)dt\,, \tag{0.1}$$

where x and y are complex variables. Here C is a path of integration such that the integrand vanishes at its terminal points. The Airy function

$$u(x) = \int_C \exp\Big(-\frac{t^3}{3} - tx\Big)dt$$

may be regarded as a confluent type of the Gauss hypergeometric function $F(\alpha, \beta, \gamma, x)$ ([3]). There exists an analogous relation between (0.1) and the Appell hypergeometric function $F_1(\alpha, \beta, \beta', \gamma, x, y)$ ([2]). The function (0.1) is called Pearcey's integral or an Airy function of two variables (cf.[4]). The integral $z_C(x, y)$ satisfies a system of partial differential equations of the form

$$\begin{aligned} \partial_x{}^2 u &= \frac{x}{2}\partial_x u + \frac{y}{4}\partial_y u + \frac{1}{4}u\,, \\ \partial_x\partial_y u &= \frac{x}{2}\partial_y u + \frac{y}{4}u\,, \\ \partial_y{}^2 u &= 2\partial_x u\,, \end{aligned} \tag{0.2}$$

whose solutions constitute a 3-dimensional vector space over $\mathbf{C}$ (cf.[2]). This system is equivalent to

$$dV = (P(x, y)dx + Q(x, y)dy)V \tag{0.3}$$

Received December 25, 2002