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# Dedekind Sums with Roots of Unity and Their Reciprocity Law

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## 1. Introduction

For positive integers $k$ and $h$ with $\gcd(k, h) = 1$, the classical Dedekind sum $s(k, h)$ is defined by

$$s(k,h) = \sum_{a=1}^{h-1} \bar{B}_1\left(\frac{a}{h}\right)\bar{B}_1\left(\frac{ka}{h}\right),$$

where $\bar{B}_1(x)$ is the first Bernoulli function. For $x$ real, the $n$-th Bernoulli function $\bar{B}_n(x)$ is defined by

$$\bar{B}_n(x) = B_n(\{x\}) \quad \text{if } n > 1 \quad \text{and} \quad \bar{B}_1(x) = \begin{cases} B_1(\{x\}) & \text{if } x \notin \mathbf{Z} \\ 0 & \text{if } x \in \mathbf{Z} \end{cases},$$

where $\{x\}$ denotes the fractional part of $x$. The most famous property of Dedekind sum is the reciprocity law

$$s(k,h) + s(h,k) = \frac{1}{12}\left(\frac{k}{h} + \frac{h}{k} + \frac{1}{kh}\right) - \frac{1}{4}$$

(see [6]). Various people generalized this sum and obtained their reciprocity laws.

In [5] we focused on Apostol's generalized sum

$$s_n(k,h) = \sum_{a=1}^{h-1} \frac{a}{h}\bar{B}_n\left(\frac{ka}{h}\right),$$

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