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The Modified Jacobi-Perron Algorithm over $\mathbf{F}_q(X)^d$

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Introduction

The Jacobi-Perron type algorithms are some kinds of multi-dimensional continued fraction algorithms, which have been studied by many authors in the case of real numbers. The following map S is associated to the Jacobi-Perron algorithm:

$$S(x_1, x_2, \ldots, x_d) = \left(\frac{x_2}{x_1} - \left[\frac{x_2}{x_1}\right], \ldots, \frac{x_d}{x_1} - \left[\frac{x_d}{x_1}\right], \frac{1}{x_1} - \left[\frac{1}{x_1}\right]\right)$$

for $(x_1, x_2, \ldots, x_d) \in [0,1]^d$. Then the ergodic properties of S give some metric results of the Jacobi-Perron algorithm. We refer to [12] for the real number case. In this paper, we consider a modified version of this algorithm, which is called Brun's algorithm, over formal power series.

Let $\mathbf{F}_q$ be a finite fields with q elements and we consider the following:

$\mathbf{F}_q[X] = \{a_n X^n + a_{n-1} X^{n-1} + \cdots + a_1 X + a_0 \,, a_i \in \mathbf{F}_q,\ 0 \le i \le n\}$
: the set of polynomials of $\mathbf{F}_q$-coefficients,

$\mathbf{F}_q(X) = \{\frac{P}{Q} :\ P, Q \in \mathbf{F}_q[X],\ Q \neq 0\}$
: the set of rational functions,

$\mathbf{F}_q((X^{-1})) = \{a_n X^n + a_{n-1} X^{n-1} + \cdots,\ a_i \in \mathbf{F}_q,\ i \le n,\ a_n \neq 0,\ n \in \mathbf{Z}\}$
: the set of formal Laurent power series of $\mathbf{F}_q$-coefficients.

We regard $\mathbf{F}_q[X]$, $\mathbf{F}_q(X)$, and $\mathbf{F}_q((X^{-1}))$ as the set of integers, of rational numbers, and of real numbers, respectively. Then we consider the set of formal Laurent power series of negative degree as the unit interval and can define the map "S" in the same way. We call the algorithm together with S the Jacobi-Perron algorithm over $\mathbf{F}_q(X)^d$. This algorithm has been studied by Paysant-Leroux and Dubois [9], [10], Feng and Wang [2] and Inoue [4]. Indeed, they showed the convergence of the expansion ([2], [9] and [10]), some simple metric properties ([9] and [10]) and the exponential convergence ([4]). In this paper, we modify this

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