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Cutting and Pasting of Families of Submanifolds Modeled on $\mathbf{Z}_2$-Manifolds

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Introduction

All manifolds considered in this paper are in the smooth category, and they are all unoriented, with or without boundary. $\mathbf{Z}_2$ denotes the cyclic group of order 2.

We will consider families of submanifolds of a manifold, and define the SK-group of such families. We will investigate the relationship between the SK-group of families and the SK-group of $\mathbf{Z}_2$-manifolds.

Let $m \geq 0$ be an integer. Let P and Q be m-dimensional compact manifolds with boundary ∂P and ∂Q, respectively, and $\varphi : \partial P \to \partial Q$ be a diffeomorphism. Pasting P and Q along the boundary by φ, we obtain a closed manifold $P \cup_\varphi Q$. For another diffeomorphism $\psi : \partial P \to \partial Q$ we obtain another closed manifold $P \cup_\psi Q$. The two closed manifolds $P \cup_\varphi Q$ and $P \cup_\psi Q$ are said to be *obtained from each other by cutting and pasting* (*S*chneiden und *K*leben in German). Two m-dimensional closed manifolds M and N are said to be SK-*equivalent* to each other, if there is an m-dimensional closed manifold L such that the disjoint union $M + L$ is obtained from $N + L$ by a finite sequence of cuttings and pastings. This is an equivalence relation on $\mathfrak{M}_m$, the set of m-dimensional closed manifolds. Note that if M and N are SK-equivalent then $\chi(M) = \chi(N)$ since

$$\chi(P \cup_\varphi Q) = \chi(P) + \chi(Q) - \chi(\partial P) = \chi(P \cup_\psi Q),$$

where χ denotes the Euler characteristic. Denote by $[M]$ the equivalence class represented by M, and by $\mathfrak{M}_m/SK$ the quotient set of $\mathfrak{M}_m$ by the SK-equivalence. $\mathfrak{M}_m/SK$ becomes a semigroup with the addition induced from the disjoint union of manifolds. The Grothendieck group of $\mathfrak{M}_m/SK$ is called the SK-*group* of m-dimensional closed manifolds and is denoted by SK_m. This group has been introduced and observed by Karras, Kreck, Neumann and Ossa [7]. Note that $[M] = [N]$ in SK_m if and only if M, N are SK-equivalent to each other.

Let $\mathfrak{M}_m^{\mathbf{Z}_2}$ be the set of m-dimensional closed $\mathbf{Z}_2$-manifolds. Taking $\mathbf{Z}_2$-equivariant diffeomorphisms as pasting diffeomorphisms, we can perform $\mathbf{Z}_2$-equivariant cuttings and pastings in $\mathfrak{M}_m^{\mathbf{Z}_2}$ in a similar way as in $\mathfrak{M}_m$, and define an SK-equivalence relation on $\mathfrak{M}_m^{\mathbf{Z}_2}$. Then we

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