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On the Structure of Strictly Complete Valuation Rings

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Introduction

The purpose of this paper is to determine the structure of strictly complete discrete valuation rings of finite dimension with equal characteristic, and moreover study the finite extensions of these rings. First, we begin with recalling the definition of linear topologies associated with valuation rings and complete valuation rings (see [7]).

For a valuation ring A, we consider the linear topology on QA with fundamental system of neighborhoods of 0 :

$$\Sigma_A = \{a\mathfrak{m}(A) \mid a \in A,\ a \neq 0\}\,.$$

This topology is said to be the A-topology on QA. Here QA is the quotient field of A and $\mathfrak{m}(A)$ is the unique maximal ideal of A.

For a valuation ring A, the completion

$$\hat{A} = \text{proj.}\lim A/\mathfrak{a} \quad (\mathfrak{a} \in \Sigma_A)$$

with respect to the A-topology is also a valuation ring (see [7, Theorem 1]). If $A \cong \hat{A}$ holds naturally, then the valuation ring A is said to be complete. Here we introduce the notion of strictly complete valuation rings as follows:

DEFINITION. A valuation ring A is said to be strictly complete, if the valuation rings $A/\mathfrak{p}$ are complete for any $\mathfrak{p} \in Spec\ A$.

For valuation rings of dimension one, the strictly completeness is equivalent to the completeness. However, for $n \geqq 2$, there exists a complete valuation ring of dimension n, which is not strictly complete. See Example 2. The main results are stated as follows:

THEOREM 1. *Let A be an equal characteristic strictly complete discrete valuation ring of dimension n and $K = QA$.*

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