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Hausdorff Dimension of a Cantor set on R^1

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1. Introduction

It is well-known that the ergodic properties of one dimensional dynamical system are determined by the spectrum of the Perron-Frobenius operator associated with transformations (for example [2], [3]). In [4] and [5], constructing renewal equations on symbolic dynamics, Mori defined Fredholm matrix $\Phi(z)$. He proved that the determinant $\det(I - \Phi(z))$ plays similar role as the Fredholm determinant of nuclear operators, that is, the zeros of the determinant are the reciprocals of the eigenvalues of the Perron-Frobenius operator. So he also call $\det(I - \Phi(z))$ the Fredholm determinant. He also showed the reciprocal of the Fredholm determinant equals the dynamical zeta function.

Using this idea, constructing α-Fredholm matrix, Mori determined the Hausdorff dimension of Cantor sets generated by piecewise linear transformations on intervals, and studied the ergodic properties of the dynamical system on Cantor sets ([8]).

In [9], he also calculated the Hausdorff dimension of Cantor sets on a plane generated by Koch-like mappings or Sierpinskii-like mappings, using the spectra of the Perron-Frobenius operator associated with piecewise linear mappings on a plane (cf. [7]).

For Cantor sets generated by transformations which are not necessarily piecewise linear, one considers $\log |F'|$ as potential, and approximates a transformation F by piecewise linear transformations on symbolic dynamics. Then zeta functions corresponding to these piecewise linear transformations converges to that of F. Using this fact, Mori ([6]) estimated the Hausdorff dimension of Cantor sets generated by piecewise C^2 and Markov transformations. Jenkinson and Pollicott ([10]) also estimated the Hausdorff dimension of the Cantor set generated by continued fraction expansion in a similar way.

In this article, we will consider a Cantor set generated by a piecewise linear and Markov transformation on $\mathbf{R}^1$ with parameter p. Roughly speaking, our Cantor set is the set of points which returns infinitely many times. We will consider the Hausdorff dimension of this Cantor set as a function of p. If $0 < p < 1/4$, Markov chain is recurrent, hence the Hausdorff

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