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Large Time Behavior and Uniqueness of Solutions of a Weakly Coupled System of Reaction-Diffusion Equations

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1. Introduction

We consider nonnegative solutions of the initial value problem for a weakly coupled system

$$(1)\qquad \begin{cases} \dfrac{\partial u_i(x,t)}{\partial t} = \Delta u_i(x,t) + u_{i+1}^{p_i}(x,t)\,, & x \in \mathbf{R}^d,\ t > 0,\ i \in N^*\,, \\ u_i(x,0) = u_{i,0}(x)\,, & x \in \mathbf{R}^d,\ i \in N^*\,, \end{cases}$$

where $N \geq 1$, $N^* = \{1, 2, \cdots N\}$, $d \geq 1$, $p_i > 0$ ($i \in N^*$) and $u_{i,0}$ ($i \in N^*$) are nonnegative bounded and continuous functions. Throughout this paper we mean $u_{N+i} = u_i$, $u_{N+i,0} = u_{i,0}$, $p_{N+i} = p_i$ for each $i \in \mathbf{Z}$ and $u = (u_1, u_2, \cdots, u_N)$, $u_0 = (u_{1,0}, u_{2,0}, \cdots, u_{N,0})$.

Problem (1) has a nonnegative and bounded solution at least locally in time (see Theorem 2.1). For any given initial value u_0, let $T^* = T^*(u_0)$ be the maximal existence time of the solution. If $T^* = \infty$, it is called a global solution. On the other hand, if $T^* < \infty$, there exists $i \in N^*$ such that

$$(2)\qquad \limsup_{t \to T^*} \|u_i(t)\|_\infty = \infty\,.$$

When (2) holds, we say that the solution blows up in a finite time.

Since the pioneering work of Fujita [6], the blow up and global existence of solutions to weakly coupled semilinear parabolic systems have been studied by several authors ([2], [3], [4] and [7]).

In the previous paper ([8]), we have considered the case $p_i \geq 1 (i \in N^*)$, $p_1 p_2 \cdots p_N > 1$ and proved the following results.

(I) If $2 \max_{i \in N^*}\{1 + p_i + p_i p_{i+1} + \cdots + p_i p_{i+1} \cdots p_{i+N-2}\} \geq d(p_1 p_2 \cdots p_N - 1)$, then $T^* < \infty$ for every nontrivial solution $u(t)$ of (1);

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