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A Formula for the A-Polynomials of $(-2, 3, 1+2n)$-Pretzel Knots

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1. Introduction

Let M be a compact 3-manifold such that ∂M is a torus and $\{\lambda, \mu\}$ a basis of $\pi_1(\partial M)$. Then $R = \mathrm{Hom}(\pi_1(M), \mathrm{SL}(2, \mathbf{C}))$ is an affine algebraic variety. Let R_U be the set of representations $\rho \in R$ such that

$$\rho(\lambda) = \begin{pmatrix} l & * \\ 0 & 1/l \end{pmatrix} \qquad \rho(\mu) = \begin{pmatrix} m & * \\ 0 & 1/m \end{pmatrix}$$

for some $l, m \in \mathbf{C}$. Note that any element of R can be conjugated to such a representation because λ and μ are commutative and that the Zariski closure of the image of the eigenvalue map $\xi : R_U \to \mathbf{C}^2$ defined by $\xi(\rho) = (l, m)$ is an algebraic subset of $\mathbf{C}^2$. Let $C_1, C_2, \cdots, C_k$ be the one-dimensional components of the closure of $\xi(R_U)$ and $g_1(l, m), g_2(l, m), \cdots, g_k(l, m) \in \mathbf{Z}[l, m]$ their defining polynomials which are supposed to be reduced. Then, the A-polynomial of M is defined by

$$A_M(l, m) = g_1(l, m)g_2(l, m) \cdots g_k(l, m)\,.$$

When M is the complement of a knot K in S^3, we choose $\{\lambda, \mu\}$ as the pair of the preferred longitude and the meridian of K. Then, the A-polynomial always has a factor $l - 1$, and so we shall compute $A_K(l, m) = A_M(l, m)/(l - 1)$.

In the study of knot theory, the polynomial invariants, such as Alexander and Jones polynomials, are very much useful and have been evaluated for a large number of knots. However, the A-polynomials have been computed for only some simple knots, see [1]. In particular, except for torus knots, there had been no formulae for the A-polynomials of infinite series of knots until Hoste and Shanahan found formulae for two infinite families of 2-bridge knots, including twist knots, in [3].

Inspired by [3], in this paper, we will derive a formula for the A-polynomials of the $(-2, 3, 1+2n)$-pretzel knots. Let K_n denote the $(-2, 3, 1+2n)$-pretzel knot depicted in Figure 1, where n is the number of left-handed full twists contained in the box. Note that

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