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A Note on Local Reduction Numbers and a^*-Invariants of Graded Rings

Duong Quôc Việt

Hanoi University of Technology

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1. Introduction

Let $(A, \mathfrak{m})$ be a Noetherian local ring of dimension d with an infinite residue field, and let I be an ideal of A. Denote by $r(I)$ the *reduction number* of I, by $\ell(I)$ the *analytic spread* of I. An interesting question is the relationship between the Cohen-Macaulay (CM) property of the Rees algebra $R(I) := \bigoplus_{n\geq 0} I^n$ and the associated graded ring $G(I) := \bigoplus_{n\geq 0}(I^n/I^{n+1})$. In the case that A is a CM ring, one approach to this problem was taken first by Goto-Shimoda when I is $\mathfrak{m}$-primary [5] in 1979. The theorem of Goto and Shimoda states:

THEOREM 1.1 [5]. *Let $(A, \mathfrak{m})$ be a CM ring of dimension d with infinite residue field. Let I be an $\mathfrak{m}$-primary ideal. Then $R(I)$ is CM iff $G(I)$ is CM and $r(I) \leq d - 1$.*

Next, other authors extended this theorem to ideals having small analytic deviation, see, e.g., [2, 4, 6]. But the most general result was obtained by Johnston-Katz in [9, Theorem 2.3], independently, Aberbach-Huneke-Trung in [1, Theorem 5.1]. Moreover, [1] also gave a similar characterization for the Gorenstein property of $R(I)$ [1, Theorem 5.8]. The method used in [1] is the study of the relationship between the so-called *local reduction numbers* of an ideal I and the a-invariant of $G(I)$ in the case that $G(I)$ is CM.

Set $L_i(I) := \{I \subseteq \mathfrak{p} \in \text{ Spec } A \mid \ell(I_\mathfrak{p}) = \text{ht}(\mathfrak{p}) \leq i\};\ i \leq \ell(I)$. The number

$$r_i(I) = \begin{cases} i - \text{ht}(I) & \text{if } i < \text{ht}(I) \\ \max\{r(I_\mathfrak{p}) - \text{ht}(\mathfrak{p}) \mid \mathfrak{p} \in L_i(I)\} + i & \text{if } \text{ht}(I) \leq i \leq \ell(I) \end{cases}$$

is called the *$i-th$ local reduction number* of I (see [1]). These invariants have been shown to play an important role in studying the CM and Gorenstein property of Rees algebras, see, e.g., [1] and [15]. Note that Aberbach-Huneke-Trung's method in [1] yielded important information concerning the local reduction numbers of an ideal I and a-invariant of $G(I)$ in the case that $G(I)$ is CM. The aims of [1] were achieved by the following result.

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