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On the Interval Maps Associated to the α-mediant Convergents

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1. Introduction

For an irrational number $x \in (0,1)$, if a non-zero rational number $\frac{p}{q}$, $(p,q)=1$, satisfies $\left|x-\frac{p}{q}\right| < \frac{1}{2q^2}$, then it is the nth regular principal convergent $\frac{p_n}{q_n}$ for some $n \geq 1$. Here, the nth regular principal convergents are defined by the regular continued fraction expansion of x:

$$x = \frac{1|}{|a_1} + \frac{1|}{|a_2} + \frac{1|}{|a_3} + \cdots .$$

We put

$$\begin{cases} p_{-1} = p_{-1}(x) = 1\,, & p_0 = p_0(x) = 0 \\ q_{-1} = q_{-1}(x) = 0\,, & q_0 = q_0(x) = 1 \end{cases}$$

and

$$\begin{cases} p_n = p_n(x) = a_n \cdot p_{n-1} + p_{n-2} \\ q_n = q_n(x) = a_n \cdot q_{n-1} + q_{n-2} \end{cases} \quad \text{for} \quad n \geq 1\,.$$

Then it is well-known that

$$\frac{p_n}{q_n} = \frac{1|}{|a_1} + \frac{1|}{|a_2} + \cdots + \frac{1|}{|a_n} \qquad \text{for} \quad n \geq 1\,.$$

If $x \in [k, k+1)$ for an integer k, we define its nth regular principal convergent by $\frac{p_n(x-k)}{q_n(x-k)} + k = \frac{p_n(x-k) + k \cdot q_n(x-k)}{q_n(x-k)}$.

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