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On Abelian p-Extensions of Formal Power Series Fields

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Introduction

Let p be a prime number. Then a field k is said to be p-quasifinite, if k is a perfect field of characteristic p and $\mathrm{Gal}(k_{sep}^{[p]}/k) \cong \mathbf{Z}_p$. Here $k_{sep}^{[p]}$ is the maximal separable p-extension of k and $\mathbf{Z}_p$ is the ring of p-adic integers.

Suppose that k is a p-quasifinite field, $n \geqq 1$ and $K = k((t_n)) \cdots ((t_1))$ is a formal power series field in n variables with coefficient field k. Then the nth Milnor K-group $K_n^M K$ of K gives rise to a topological group by introducing the weak topology (see §4). Moreover we put $\Gamma K = \mathrm{Gal}(K_{ab}^{[p]}/K)$, where $K_{ab}^{[p]}$ is the maximal abelian p-extension of K. Then the following results are obtained.

Main Theorem. *Let k be a p-quasifinite field, $n \geqq 1$ and $K = k((t_n)) \cdots ((t_1))$. Then*

(i) *for any element $F \in \Gamma k$ having the property $\Gamma k = F^{\mathbf{Z}_p}$, there exists a homomorphism*

$$\rho_K : K_n^M K \longrightarrow \Gamma K$$

of topological groups which satisfies the following two conditions:

(1) *Take any finite separable p-extension K'/K of fields. Then*

$$\overline{N_{K'/K} K_n^M K'} = \rho_K^{-1}(\mathrm{Gal}(K_{ab}^{[p]}/K' \cap K_{ab}))\,.$$

Moreover, ρ_K induces an isomorphism:

$$K_n^M K / \overline{N_{K'/K} K_n^M K'} \cong \mathrm{Gal}(K' \cap K_{ab}/K)$$

of abelian groups. Here "overline" means the closure of $K_n^M K$ with respect to the weak topology.

(2) *Take any $\alpha \in K_n^M K$. Then*

$$\rho_K(\alpha)\Big|_{k_{ab}^{[p]}} = F^{\ell(\alpha)}\,.$$

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