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On the Sylow p-Subgroups of the Ideal Class Groups of Some Imaginary Cyclic Fields of Degree $p-1$

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Introduction

In this paper, we investigate how to construct imaginary cyclic fields of degree $p-1$ whose Sylow p-subgroups of the ideal class groups are not cyclic. This paper supplements our previous paper [6].

For a given integer n, Yamamoto [10] proved that there exist infinitely many imaginary quadratic fields whose ideal class groups contain $(\mathbf{Z}/n\mathbf{Z})^2$ as a subgroup. Moreover, Nakano [7] proved that there exist infinitely many algebraic number fields K of fixed degree $m = r_1 + 2r_2$ whose ideal class groups contain $(\mathbf{Z}/n\mathbf{Z})^{r_2+1}$ as a subgroup, where r_1 and r_2 denote the number of real and imaginary embeddings of K into $\mathbf{C}$, respectively. (Other related papers are those by Ishida [5] and Azuhata and Ichimura [1].) In the present paper, we give another way to construct imaginary cyclic fields of degree $p-1$ whose ideal class groups contain $(\mathbf{Z}/p\mathbf{Z})^2$ as a subgroup for a given prime p with $p \equiv 1 \pmod 4$.

Let p be a fixed odd prime. Let ζ be a primitive p-th root of unity and put $\omega := \zeta + \zeta^{-1}$. In Section 1, we give a sufficient condition for an imaginary cyclic field of degree $p-1$ containing $\mathbf{Q}(\omega)$ to have class number divisible by p (Theorem 1.1). We can easily verify whether our condition holds for given imaginary cyclic fields of degree $p-1$ or not (see Proposition 3.2). The condition which was given in our previous paper [6] is included in the present one, as we will see in (2) of Remark 1.2. In Section 2, there are two goals. The first is to prove Theorem 1.1. The second is to give infinitely many imaginary quadratic fields of degree $p-1$ containing $\mathbf{Q}(\omega)$ whose class numbers are divisible by p. This gives another proof of [6, Theorem 2]. In Section 3, we construct imaginary cyclic fields of degree $p-1$ which have ideal class groups of p-ranks greater than one in the case of $p \equiv 1 \pmod 4$. Moreover we give a parametric family of such fields (Example 3.5). Unfortunately, the author has not yet determined whether this family is infinite.

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