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On Integral Geometry in the Four Dimensional Complex Projective Space

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1. Introduction and result

The name "Integral Geometry" was introduced by W. Blaschke in his book "Vorlesungen über Integralgeometrie" and later elaborated by him and his colleagues in their subsequent papers in the 30's. Integral geometry treats integrations of geometric invariants of geometric objects such as points, lines, submanifolds and elements of transformation groups.

Let G be a Lie group and K a closed subgroup of G. If M and N are submanifolds of the Riemannian homogeneous space G/K. Then one of main topics in the present work will compute the following integral

$$\int_G \mathrm{vol}(M \cap gN) d\mu(g) .$$

The Poincaré formula means equalities which represent the above integral by some geometric invariants of submanifolds M and N of G/K. For example, in the case that G is the group of isometries of Euclidean space $\mathbf{R}^n$, and M and N are submanifolds of $\mathbf{R}^n$; then the results of above integral lead to remarkable integral formulas by Poincaré, Crofton and other integral geometers. When G is the unitary group $U(n+1)$ acting on complex projective space $\mathbf{C}P^n$, M and N are complex submanifolds of $\mathbf{C}P^n$; then the evaluation of above integral leads to the results obtained by L. A. Santaló [8] and R. Howard [4]. In the same case, if M is a totally real submanifold and N a complex one, and M, N are totally real submanifolds, then the evaluation of above integral gives the results of R. Howard [4]. The present author and H. Tasaki [5], [6] gave the Poincaré formulas of real surfaces and complex hypersurfaces of $\mathbf{C}P^n$, and of two real surfaces of $\mathbf{C}P^2$ using the Kähler angle.

Recently, H. Tasaki [10] generalized the notion of the Kähler angle. Using this generalized Kähler angle, he obtained the Poincaré formula (see Section 3) of general submanifolds, which are neither complex nor totally real submanifolds of $\mathbf{C}P^n$. Although this formula holds under the general situation, it is difficult to give an explicit description through the concrete

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