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Determinantal Expressions for Hyperelliptic Functions in Genus Three

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1. Introduction

Let $\sigma(u)$ and $\wp(u)$ be the usual functions in the theory of elliptic functions. In the paper [12] the author gave a natural generalization to the case of genus two for the two formulae

$$(-1)^{(n-1)(n-2)/2}1!2!\cdots(n-1)!\frac{\sigma(u^{(1)}+u^{(2)}+\cdots+u^{(n)})\prod_{i<j}\sigma(u^{(i)}-u^{(j)})}{\sigma(u^{(1)})^n\sigma(u^{(2)})^n\cdots\sigma(u^{(n)})^n}$$
$$=\begin{vmatrix} 1 & \wp(u^{(1)}) & \wp'(u^{(1)}) & \wp''(u^{(1)}) & \cdots & \wp^{(n-2)}(u^{(1)}) \\ 1 & \wp(u^{(2)}) & \wp'(u^{(2)}) & \wp''(u^{(2)}) & \cdots & \wp^{(n-2)}(u^{(2)}) \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & \wp(u^{(n)}) & \wp'(u^{(n)}) & \wp''(u^{(n)}) & \cdots & \wp^{(n-2)}(u^{(n)}) \end{vmatrix} \tag{1.1}$$

discovered by Frobenius and Stickelberger [8], and

$$(-1)^{n-1}(1!2!\cdots(n-1)!)^2\frac{\sigma(nu)}{\sigma(u)^{n^2}}=\begin{vmatrix} \wp' & \wp'' & \cdots & \wp^{(n-1)} \\ \wp'' & \wp''' & \cdots & \wp^{(n)} \\ \vdots & \vdots & \ddots & \vdots \\ \wp^{(n-1)} & \wp^{(n)} & \cdots & \wp^{(2n-3)} \end{vmatrix}(u) \tag{1.2}$$

found earlier than the first one in the paper of Kiepert [10].

If we set $y(u)=\frac{1}{2}\wp'(u)$ and $x(u)=\wp(u)$, then we have an equation $y(u)^2=x(u)^3+\cdots$, that is a defining equation of the elliptic curve to which the functions $\wp(u)$ and $\sigma(u)$ are attached. Here the complex number u and the coordinates $(x(u), y(u))$ correspond by the equality

$$u=\int_\infty^{(x(u),y(u))}\frac{dx}{2y}\,.$$

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